RUNOFF CURVE NUMBER METHOD: EXAMINATION OF THE INITIAL ABSTRACTION RATIO

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Abstract: The Initial Abstraction ratio (Ia/S, or λ) in the Curve Number (CN) method was assumed in its original development to have a value of 0.20. Using event rainfall-runoff data from several hundred plots this assumption is investigated, and λ values determined by two different methods. Results indicate a λ value of about 0.05 gives a better fit to the data and would be more appropriate for use in runoff calculations. The effects of this change are shown in terms of calculated runoff depth and hydrograph peaks, CN definition, and in soil moisture accounting. The effect of using λ=0.05 in place of the customary 0.20 is felt mainly in calculations that involve either lower rainfall depths or lower CNs.

INTRODUCTION

Originally developed by the Soil Conservation Service (SCS, now Natural Resources Conservation Service or NRCS) in the 1950s for internal use, the Curve Number method for estimating direct runoff from rainstorms is now widely used in engineering design, post- event appraisals, and environmental impact estimation. Background for this is found in the NRCS document National Engineering a Handbook, Section 4, "Hydrology", or "NEH-4" (SCS, 1985). The general runoff equation is

$$Q = (P-Ia)^2/(P-Ia+S) \qquad \text{for } P \geq Ia \qquad (1a)$$
$$Q = 0 \qquad \text{for } P \leq Ia \qquad (1b)$$

Where Q is the direct runoff depth, P is the event rainfall depth, Ia is an "initial abstraction" or event rainfall required for the initiation of runoff, and S is a site index defined as the maximum possible difference between P and Q as $P \rightarrow \infty$. P- Ia is also called "effective rainfall", or Pe.

All have units of length, and the equation is dimensionally homogeneous. The index S, which has the limiting values of 0 and ∞, is transformed to the more intuitive "Curve Number" by the equation CN=1000/(10+S), where S is in inches. CN, which is dimensionless, may take values from 0 to 100, is an index of the land condition as indicated by soils, cover, land use.

Though the developmental history and documentation is obscure, the relationship between Ia and S was fixed at Ia = 0.2S. Inserting that value into equation 1 gives

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$$Q = (P-0.2S)^2/(P+0.8S) \qquad P \geq 0.2S \qquad (2a)$$
$$Q = 0 \qquad P \leq 0.2S \qquad (2b)$$

The goal here is to examine the data-that supports values of the Ia/S ratio, called λ (“lambda”), and suggest accommodations for updating its role.

METHODS

Two techniques, Event Analysis and Model Fitting, were used for determining Ia/S from field data sets. These are described in the following:

Event Analysis. Here, concurrent synchronized break-point records of both rainfall and runoff depth are required. The event rainfall depth recorded when the direct runoff hydrograph begins is taken as Ia. Knowing the total event rainfall P and the direct runoff Q, equation 1a is solved for S, and the ratio simply taken Ia/S=λ. Here each event gives a separate value of λ, and the median for a large number of events is taken as the representative watershed value. This procedure is portrayed in the Figure 1.

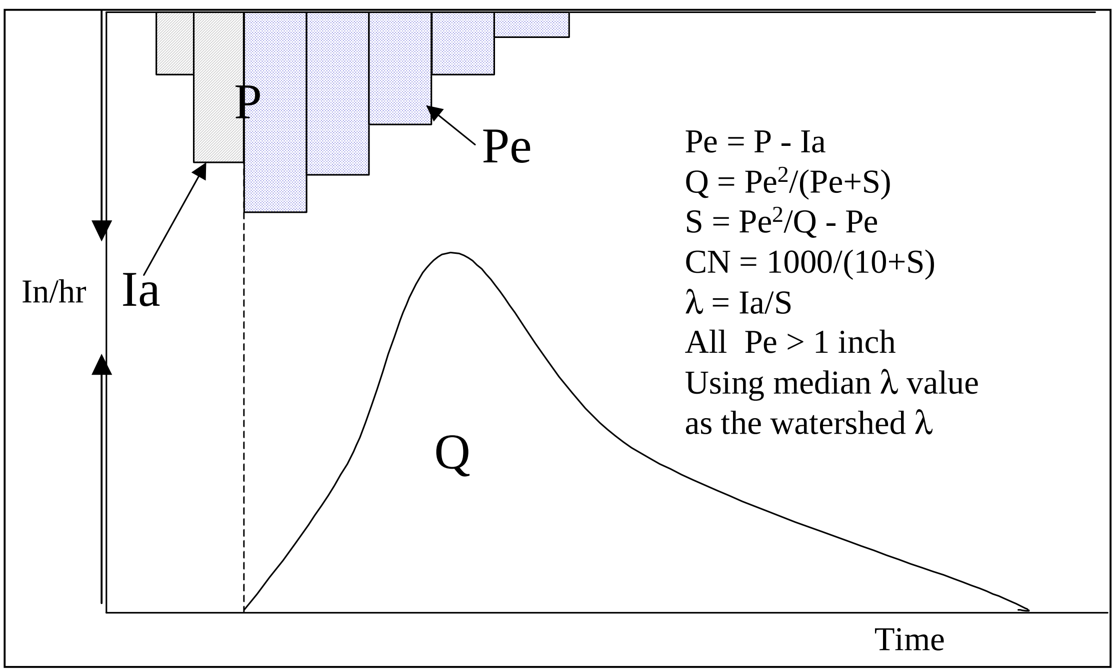

Figure 1 Event Analysis Method

General Model Fitting: Here the value of λ is simply determined by iterative least squares procedure fitting for both λ and S of the general equation.

$$Q = (P-\lambda S)^2/(P+(1-\lambda)S) \qquad \text{for } P \geq \lambda S \qquad (3a)$$
$$Q = 0 \qquad \text{for } P \leq \lambda S \qquad (3b)$$

The objective of the fitting is to find the values of λ and S such that

$$\Sigma\{Q - [(P-\lambda S)^2/(P+(1-\lambda)S)]\}^2 \quad (4)$$

is a minimum. Here each P:Q data set gives only one value of λ. An illustration of such fitting is given in Figure 2.

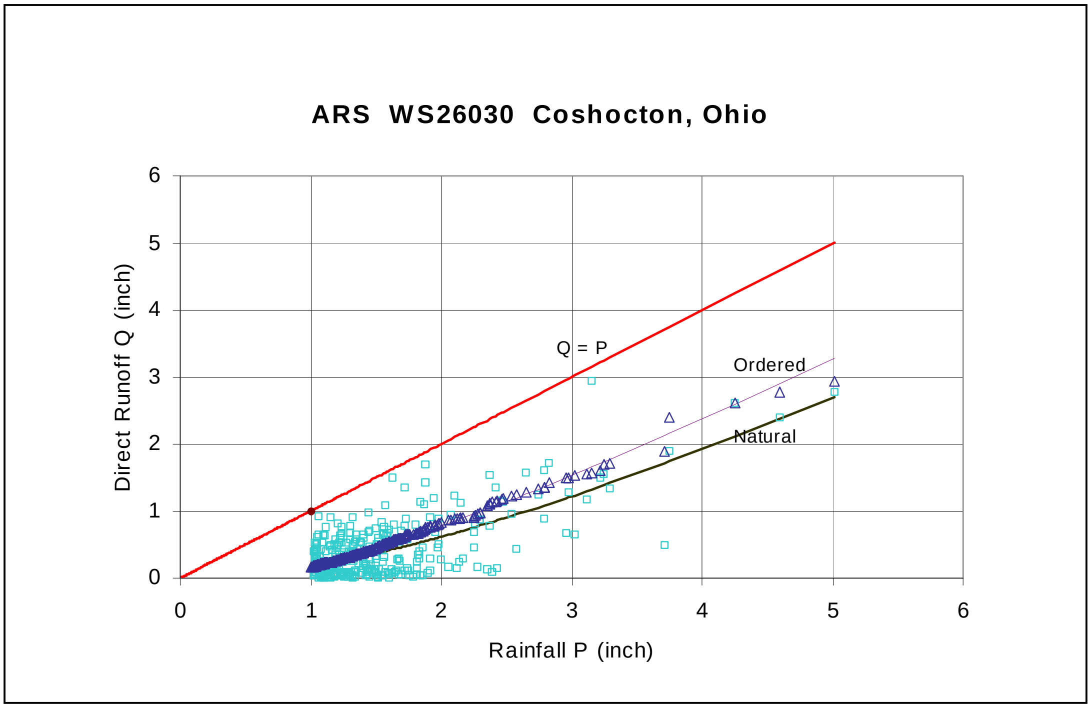

Figure 2 Model Fitting by least squares for WS26030, Coshocton Ohio

Model fitting by least squares in Figure 2 is for WS26030 located at Coshocton, OH with a drainage area of 303 acres. For the natural data (squares): S = 4.0974 inches, CN = 70.8, λ = 0.0179, R^2 = 50.50% and SE = 0.32 inch. For the ordered data (triangles): S = 2.0943 inches, CN = 82.6, λ = 0.1364, R^2 = 99.17%, and SE = 0.0372 inches.

In each of the above two methods, only "larger" storms were used. This was done to avoid the biasing effects of small storms towards high Curve Numbers. With Event Analysis, only events with Pe = P-λS ≥1 inch were used. With Model Fitting, only events with P ≥1 inch were used. As shall be seen, found values of Ia were often quite small, so that this difference between the two techniques was slight. For statistical analysis, only watersheds with more than 20 events with P ≥ 1 inch or Pe ≥ 1 inch were used.

In addition, for the model fitting determinations, both "natural" and "ordered" data sets were used. Natural data pairs the P and Q as they naturally occurred in time, and thus displays considerable variety in runoff with rainfall. Ordered data matches (usually) unnatural rank-ordered P and Q values, so that each has approximately the same return period. This is in keeping with a major application of the method, which is design work. For example, the 100-year rainfall is assumed to produce the 100-year runoff.

DATA SETS

Rainfall-runoff data from 307 watersheds or plots were used, originating from USDA-Agricultural Research Service, US Forest Service, US Geological Survey, and New Mexico State University. It covered 23 states, mainly in the east, midwest, and south of the United State. There was no data from the northwestern 1/3 of the country, from roughly California to Minnesota. A total of 28,301 events were available that met the rainfall depth (P and Pe) criteria. For event analysis, only ARS data was applicable, as it alone contained the needed detailed in-storm break-point information. All others were only rainfall and runoff depths P and Q. This is summarized in Table 1.

Table 1 Data sets and sources

Data source	# Watersheds (w) or plots (p)	Method used
ARS	134 (w)	Event Analysis, Model Fitting
USLE (ARS)	137 (p)	Model Fitting
USFS	26 (w)	Model Fitting
Jornada (NMSU)	6 (p)	Model Fitting
USGS	4 (w)	Model Fitting

These 307 watersheds all had 20 or more events which met the storm size criteria. The ARS data is available from ftp://hydrolab.arsusda.gov/pub/arswater/. The "USLE" plot data had been used in the development of the Universal Soil Loss Equation, and was downloaded from the web site: http://topsoil.nserl.purdue.edu/usle/. Forest Service data was in large part supplied in reduced form to (RHH) by Dr J.D. Hewlett of the University of Georgia, who used it in an earlier paper (Hewlett, et al., 1977; Hewlett and Fortson, 1984). The Jornada plot data, from site north of Las Cruces NM, was supplied by Dr T.J. Ward, now at the University of New Mexico. It is described in Hawkins and Ward (1998). The USGS data was supplied from local sources for a number of urban and urbanizing watersheds in the Tucson area.

RESULTS

In general, the results showed that λ is not a constant from storm to storm, or watershed to watershed, and that the assumption of λ=0.20 is unusually high.

Event Analysis: it was found that Ia/S ratios varied greatly between storms within watersheds, and also between the 134 watersheds. For each watershed the median λ was used to describe λ. The general findings are included in Table 2. Values of the found λ varied from 0.0005 to 0.4910, with a median of 0.0476. There was a distinct negative skew, or a crowd of smaller values. Over 90% were less than 0.2.

Table 2 Summary results of λ value for ARS watersheds (n = 134)

	Event Analysis	Model Fitting(natural)	Model Fitting(ordered)
Min	0.0005	0.0000	0.000
Median	0.0476	0.0001	0.0736
Mean	0.0701	0.0555	0.1491
Max	0.4910	0.5766	0.9682
STDV	0.0812	0.0983	0.2001
Skewness	2.5899	2.8364	1.8725
% ≤0.20	93.7	93.3	72

Model Fitting. As described previously, both natural and ordered data sets were fitted to the general runoff equation by least squares to determine Ia and S. Results were more varied than with Event Analysis, although this may be explained with the much larger sample size (N=307). For natural data, the λ range was from 0 to 0.996, with a median of 0, and for ordered data, the λ range was form 0 to 0.9793 with a median of 0.0618. A summary of these results is given in Table 3.

Table 3 Summary Results of λ values from model fitting

			Natural Data				Ordered Data			
	N	Total Event	Max	Mean	Median	Min	Max	Mean	Median	Min
ARS	134	12499	0.5766	0.0555	0.0001	0	0.9682	0.1491	0.0736	0
USLE	137	11140	0.996	0.0997	0	0	0.9266	0.1581	0.061	0
Others	36	4392	0.4727	0.04	0	0	0.9793	0.0992	0.0044	0
Total	307	28031	0.996	0.0734	0	0	0.9793	0.1472	0.0618	0

APPLICATIONS

From the above results, it is obvious that a more appropriate "rounded" value of Ia/S would be in about 0.05. Using this value the runoff equation is adjusted, for which a new set of CNs based on λ=0.05 must be determined.

Runoff equation: Using Ia/S=0.05 the runoff equation becomes

$$Q = (P-0.05S)^2/(P+0.95S) \qquad P\geq 0.05S \qquad (5a)$$
$$Q = 0 \qquad P\leq 0.05S \qquad (5b)$$

However, the S values in the above equation are not the same as previously used assuming Ia/S=0.20. They are defined on a system of Ia/S=λ=0.05.

<u>Equivalent CN</u>: Based on the above experiences with λ the data was fitted by least squares to the CN equation for each case: that is, for $\lambda = 0.05$ and for the traditional value of $\lambda = 0.20$. The latter is the basis for existing CN Tables. In 252 of the 307 cases (approximately 5 out of 6) the 0.05 fitting produced a higher r^2 and lower SE.

The relationships found between the values of $S_{0.05}$ and $S_{0.20}$ were, for natural and ordered data respectively:

$$S_{0.05}=1.344S_{0.20}^{\ 1.149} \qquad r^2=99.38\% \qquad (6)$$
$$S_{0.05}=1.316S_{0.20}^{\ 1.164} \qquad r^2=99.44\% \qquad (7)$$

where $S_{0.05}$ and $S_{0.2}$ are in inches. Rounded consensus values of these two almost-identical findings condense to

$$S_{0.05}=1.33S_{0.20}^{1.15} \qquad (8)$$

Preserving the basic definition of CN = 1000/(10+S), the above relationship permits conversion from the 0.20-based CNs to 0.05-based CNs. Making the substitutions and simplifying gives

$$CN_{0.05} = \frac{100}{1.879[100/CN_{0.20}-1]^{1.15} + 1} \qquad (9)$$

Consideration of equation 8 shows that $S_{0.05}=S_{0.20}$ at 0.148 inch, or $CN_{0.20}\approx98.5$. They are also equal at S=0, or CN=100. At these seldom-encountered levels we suggest they be considered equal. Table 4 gives the $CN_{0.05}$ corresponding to current values of $CN_{0.20,}$ taken from equation 9, or "Conjugate" Curve Numbers

Table 4. Conjugate Curve Numbers and P_{crit}

$CN_{0.20}$	$S_{0.20}$(in)	$CN_{0.05}$	$S_{0.05}$(in)	P_{crit} (in)
100.00	0.000	100.00	0.000	-----
95.00	0.526	94.02	0.636	2.44
90.00	1.111	86.95	1.501	1.72
85.00	1.765	79.64	2.556	1.95
80.00	2.500	72.39	3.815	2.27
75.00	3.333	65.31	5.311	2.63
70.00	4.286	58.51	7.091	3.05
65.00	5.385	52.03	9.219	4.51
60.00	6.667	45.90	11.785	4.04
55.00	8.182	40.14	14.915	4.64
50.00	10.000	34.74	18.787	5.35
45.00	12.222	29.71	23.663	6.15
40.00	15.000	25.03	29.947	7.13
35.00	18.571	20.71	38.285	8.35

Comparisons: How does the modification of Ia/S affect calculated values of runoff? First, by equating the runoff equations using 0.05 and 0.20 and making the transformation of CNs using equations 8 or 9, the rainfall depth corresponding to equal runoffs for conjugate CNs can be determined. This rainfall is shown as P_{crit} in Table 4. There is no closed solution: P_{crit} was determined numerically. For P greater than P_{crit} use of Ia/S=0.2 will result in higher calculated runoffs. For lesser P values, a higher runoff will be found using Ia/S=0.05.

Second: Figure 4 shows rainfall-runoff plots over a range of rainfalls and conjugate CN values. P_{crit} is the locus of the crossing points. As can be seen, the conjugate curves are similar at a wide range of frequently-encountered values of P and CN.

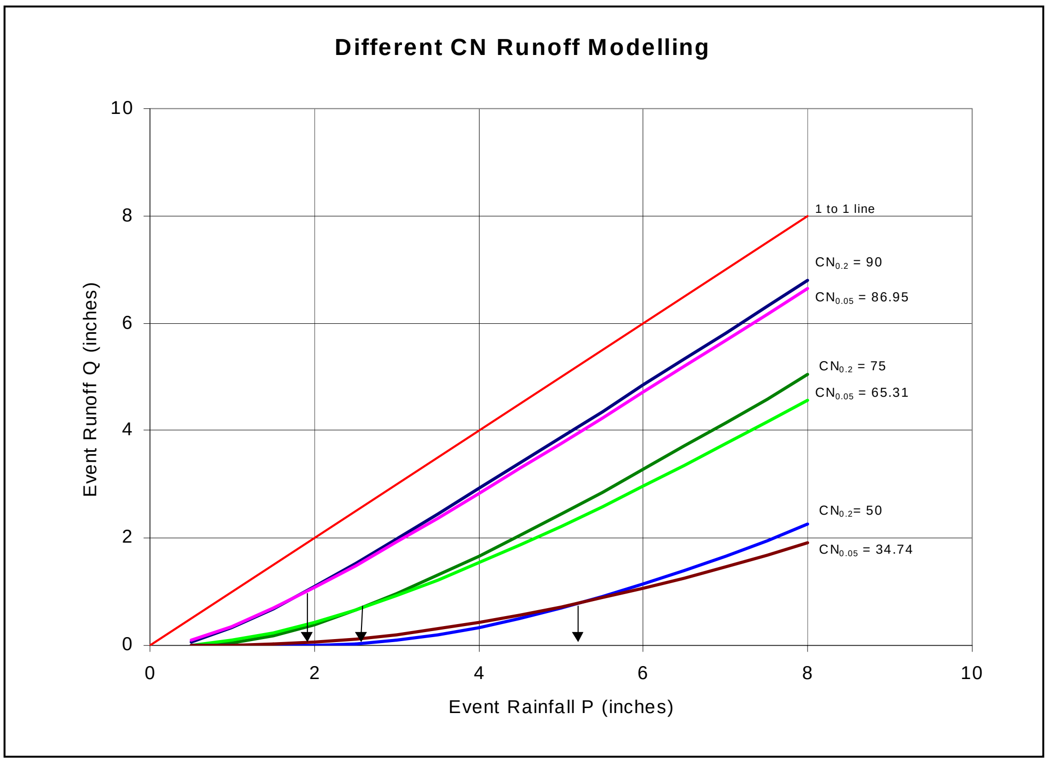

Figure 3 Rainfall and runoff for three conjugate CN pairs

Third, in modeling hydrographs similar differences are seen. Examples are shown in Figures 3 and 4. Similar hydrographs, peaks, and timing result for the higher CNs, but distinct differences are shown with the lower CN. Figure 4 shows highlights the differences for the $CN_{0.20}$=50 example. Using λ=0.05 calculates a smaller Ia, giving direct runoff earlier in the event. Here this leads to a peak about 60% increases in peak flow.

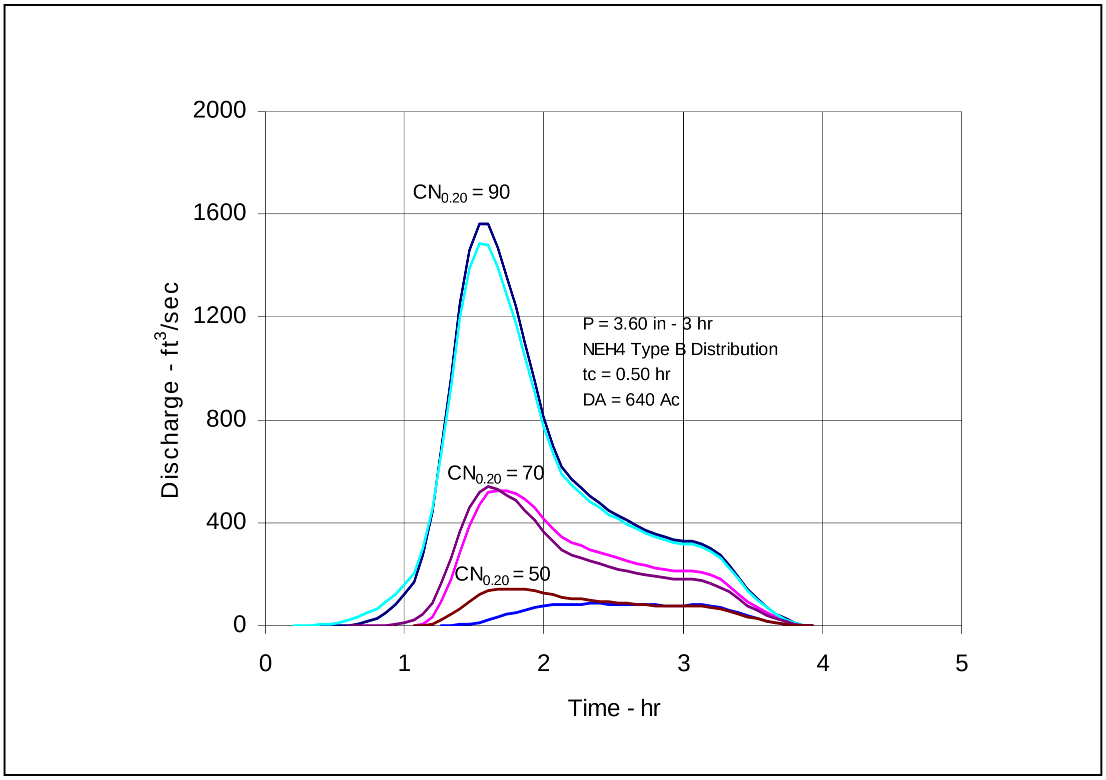

Figure 4 Comparative hydrographs for conjugate CNs .

This low CN-small storm scenario will be important in small storms and lower CNs. This situation characterizes forested watersheds, especially area of modest return period storms depths (e.g., large portions of the interior west). This condition would also prevail in continuous modeling scheme, which consider all rainfalls – usually on a daily basis – regardless of size. Differences also exist at higher rainfall extremes, but have not been evaluated here. The major effect seems to be at lower CNs at lower rainfalls, or in general at low P/S situations.

CONCLUSIONS

Ia/S: As determined by two separate methods, Ia/S (or λ) of 0.05 fits observed rainfall-runoff data much better than does the handbook value of 0.20.

Runoff Equation: With λ=0.05, the runoff equation becomes

$$Q = (P-0.05S_{0.05})^2/(P+0.95S_{0.05}) \qquad P \geq 0.05S_{0.05}$$
$$Q = 0 \qquad P \leq 0.05S_{0.05}$$

Change of CN: Altering λ requires a change of handbook CNs. That is, if λ is changed, a different CN must be used. The relationship is

$$S_{0.05} = 1.33S_{0.20}^{1.15}$$

(where S is in inches) relates the two conditions. It was determined from data from 307 watersheds. New handbook CN tables for λ=Ia/S=0.05 might be constructed using this relationship.

<u>Differences</u>: The most obvious differences in runoff modeling are at lower CNs and lower rainfalls, or in general at low P/S situations. This would prevail for low CN watersheds, more frequently occurring rainstorm depths, and for climates where the more modest design return period rainfalls are found.

It is interesting to note that Victor Mockus father of the runoff curve number method in a conversation with Professor V. M. Ponce (Ponce 1996) indicated that he would not oppose changing the Ia/S ratio if the data warranted it.

FUTURE ACTIONS

The ARS/NRCS Curve Number work group has developed a plan of work for the implementation of the Ia/S ratio of 0.05 into the NRCS central record system. This will involve changing all appropriate documents, computer programs and notification of developers of computer programs that include the NRCS runoff system. It may also involve notification of local governments that include NRCS runoff system in their rules and regulations. NRCS has begun to follow the recommended plan of work. It is estimated that it might be up to five years before the revised Ia/S ratio is officially implemented.

ACKNOWLEDGEMENTS

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