# A SURFEIT OF CYCLES

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**ABSTRACT**
Chaotic sets are organized about "skeletons" of periodic orbits in the sense that every point on a chaotic set is arbitrarily close to such an orbit. The orbits have the stability property of saddles: attracting in some directions; repelling in others. This topology has implications for changing climates that evidence pronounced variability on time scales ranging from decades to tens of thousands of years. Among these implications are the following: 1. A wide range of periodicities should be (and are) observed. 2. Periodicities should (and do) shift – often abruptly – as the evolving climatic trajectory sequentially shadows first one periodic orbit and then another. 3. Models that have been "tuned" (parametric adjustment) to fit trajectorial evolution in the vicinity of one periodic orbit are likely to fail when the real system moves to another region of the phase space. 4. In response to secular forcing, chaotic sets simplify via the elimination of periodic orbits. If one accepts the reality of anthropogenic warming, the long-term prediction is loss of intrinsic variability. 5. In response to periodic forcing, nonlinear systems can manifest subharmonic resonance *i.e.*, "cyclic" behavior with periods and rotation numbers rationally related to the period of the forcing. Such cycling has been implicated in millennial and stadial variations in paleoclimatic time series. 6. Generically, the dynamics of system observables, such as climate sensitivity, are qualitatively equivalent to those of the whole. If the climate is chaotic, so too is sensitivity. These considerations receive minimal attention in consensus views of climate change that emphasize essentially one-to-one correspondence between global temperatures and exogenous forcing. *Caveat emptor*.

> "To every thoughtful naturalist the question must arise, What are these for? What have they to do with the great laws of creation? Do they not teach us something of the system of Nature? … what do these … apparent imperfections mean?"

So wrote Alfred Russel Wallace *[1855]* regarding rudimentary organs, which, under the then prevailing theory of Special Creation *[Eiseley, 1961]*, were inexplicable. Contemporary climatologists might profitably pose a similar set of questions, substituting the word "cycles" for "rudiments." As noted by Huybers and Curry

*[2006]*, the climatological record is rife with variability. On time scales of tens to hundreds of years, one has the AMO, PDO, ENSO, *etc*. *[Barnston and Livezey, 2007; Keenlyside, et al. 2008]*; over longer periods, millennial periodicities (Medieval warm period, Little Ice Age, present-day warming *[Braun et al., 2005; Grosjean et al. 2007]*) and, on stadial time scales, the advance and retreat of the ice *[Imbrie and Imbrie, 1980; Huybers and Wunsch, 2005]*. Often, these "cycles" are discussed with reference to "bifurcation" and "regime shift" *[Christiansen, 2003; Crowley and Hyde, 2008]*, terms that recall the "frozen earth," "no-ice" equilibria of simple energy balance models *[Covey, 1989]*.

**Meridional Overturning Circulation (MOC).** Writing in the *News and Views* section of *Nature*, Richard Wood *[2008]* recently commented on natural variability and climate. "Climate change," he declared

> "is often viewed as a phenomenon that will develop in the coming century. … A confounding factor is that, on these [decadal][1] timescales, … warming will not be smooth; instead, it will be modulated by natural climate variations."

Turning to a paper *[Keenlyside, et al. 2008]* (see also, *Smith et al. 2007]*) that appeared in the same issue, Wood continued:

> "The authors … predict that the MOC will weaken over the next decade, with a resultant cooling effect on climate around the North Atlantic. Such a cooling could temporarily offset the longer-term warming trend from increasing levels of greenhouse gases in the atmosphere."

MOC refers to ocean currents, such as the Gulf Stream that deliver the warmth of tropical waters to higher latitudes, thereby ameliorating boreal climates, "There is evidence," Wood observed,

> "that the strength of this circulation can fluctuate naturally over periods of decades *[Knight et al., 2005] …* ."

The anticipated cooling to which Wood refers arguably *[Goddard, 2008]* began some ten years ago, and it stands in marked contrast to "consensus" predictions *[Hansen et al. 2006; IPCC, 2007; Weart, 2008]* of unabated warming. This brings us back to Wallace's broader questions: "Why should they [climate cycles] be?" "What does their existence tell us?"

I propose the following answers:

1. Climatal cycles are blurred images, fingerprints, if you will, of the non-stable periodic orbits about which chaotic motion is organized.

[1] Square brackets in quotes denote author's clarifications.

2. A worldview that equates variability to error bars is risky; policy recommendations derived therefrom, fraught with disastrous potential.

To a greater or lesser extent, these assertions are appreciated by nonlinear dynamicists with an interest in climate – *e.g.*, *[Elsner and Tsonis, 1993; Rahmsdorf, 1995; van Veen at al. 1998; Corti et al., 1999; Lindzen et al. 2001; Crommelin, 2002; Rial et al, 2004; Braun et al. 2005; Huybers and Wunsch, 2005; Farnetio and Killworth, 2005; Huybers and Curry, 2006; Tsonis et al. 2007]*. What follows is therefore addressed principally to the larger climatological community, and especially to modelers whose current infatuation is massive "unified earth system models" *[McGuffie and Henderson-Sellers, 2005]* running on the world's fastest computers.[2] As noted by van Veen *et al. [1998],* such models can only be

> "analysed [*sic*] statistically, as they are out of reach of the ordinary analysis of dynamical systems theory."

To this, one might add that when the results of a calculation can only be comprehended through recourse to statistics, one's understanding of the problem is possibly incomplete.

**Climate Dynamics.** By inspection, the earth's climate is neither equilibrial nor periodic. Quasi-periodicity (two or more incommensurate frequencies) on stadial time scales – the result of Milankovitch forcing – is at best approximate. In the first place, orbital frequencies and their harmonics account for only a fraction of the observed variance, in which regard it has been suggested *[Huybers and Wunsch, 2005]* that only variations in obliquity are statistically significant. Additionally, stadial periodicities drift with the passage of time *[Crowley and Hyde, 2005]*. The remaining alternative is chaos, which is most often discussed *[Lorenz, 1964; Dix and Hunt, 1995; McWilliams, 2007]* with reference to sensitivity to initial conditions (SIC). SIC refers to the amplification of small initial differences. It follows that given *any* uncertainty in one's estimate of a chaotic system's initial state, the only thing that can be predicted for the long term is a probability distribution.

SIC is important, of course. It is the reason climate modelers average over multiple runs and policy makers over multiple assumptions *[McWilliams, 2007]*. Less widely appreciated, but equally important, are shifting periodicities. The latter result from the fact that chaotic sets are organized about infinities of non-stable cycles *[Auerbach et al. 1987; Gunaratne and Procaccia, 1987]*. By "organized," I mean that every point on a chaotic set is arbitrarily close to such a cycle *[Devaney, 1987];* by "non-stable," that the cycles have the stability character of saddles: attracting in some directions; repelling in others. Metaphorically, chaotic motion can be viewed as an elaborate choreography wherein the prima ballerina (the evolving trajectory) dances with a succession of partners (the aforementioned cycles), only to abandon each in his or her

[2] Sixty years ago, J. L. Borges *[1946]*, who doubtless would have been amazed by contemporary excesses *in silico*, addressed the utility of detailed simulation magnificently in his less-than-a-page parable on cartography.

turn. More precisely, periodic orbits are approached via their stable manifolds and temporarily shadowed before the trajectory moves off in the direction of their unstable manifolds. Sometimes, a small number of cycles – those strongly attracting in the stable direction and weakly repelling in the unstable – dominate. With only one cycle dominant, the result is episodes of statistical periodicity intermixed with apparently aperiodic behavior. When two or more cycles dominate, the phenomenology is "regime shift."

**The Geometry of Chaos.** Figures 1a and 1b visualize a slice (actually a stroboscopic[3] portrait, which, in systems subject to periodic forcing, is equivalent) of a chaotic attractor and a few of the cycles about which it is organized. Regarding these pictures, we make the following observations:

1. The dynamics (Figure 1a) are extraordinarily complex.
2. Embedded in the attractor are periodic orbits (Figure 1b), which because the dynamics are viewed in section, appear as points (the crosses) Only the smallest fraction of these cycles, *i.e.*, those of lowest period, are shown. In fact, the number of cycles is countably infinite, ranging from a minimum period of 7 simulated years on up to infinity.
3. The geometry is representative of chaotic attractors that arise in a variety of physical, chemical and biological contexts. Figures 1a and 1b were generated by a two-variable model (ordinary differential equations) of predator-prey interaction subject to seasonality. But take a plane pendulum with modest amounts of friction, subject it to periodic forcing and you get a qualitatively equivalent picture. Delete the friction, and you get the conservative chaos of three or more planetary bodies.

Important attributes of a chaotic climate are thus the existence of multiple periodicities and, with the passage of time, unpredictable shifts from one dynamical regime to another. As emphasized by Tsonis *et al*. *[2007]* and others, such shifts are observed in historical time series and paleoclimatic proxies.

As a consequence, evolving climates may be full of surprises *[Schneider, 2004; Tsonis et al. 2007]*. The anomalies are likely to be all the more unexpected if expectations are based on "all but the kitchen sink" simulations that have been "tuned" to historical data. In such cases, parametric adjustment fits the model to particular regions of the phase space. When the real system moves to another region, the most likely result is model failure.

**Destruction of Periodic Orbits.** If one progressively increases the intensity of some friction-like parameter, chaotic attractors simplify via the loss of periodic orbits *[King and Schaffer, 1999; Schaffer and Bronnikova, 2007]* until, at last, the behavior is equilibrial or, in the presence of periodic forcing, periodic, with the oscillations slaved to the inputs. Increasing the intensity of secular forcing, for example by adding a wind of constant velocity to the periodically forced pendulum, has the same effect. In the latter case, all behaviors are eventually lost save for rotation in the direction of the wind.

[3] By "stroboscopic," I mean viewing the dynamics at time intervals equal to the period of external forcing.

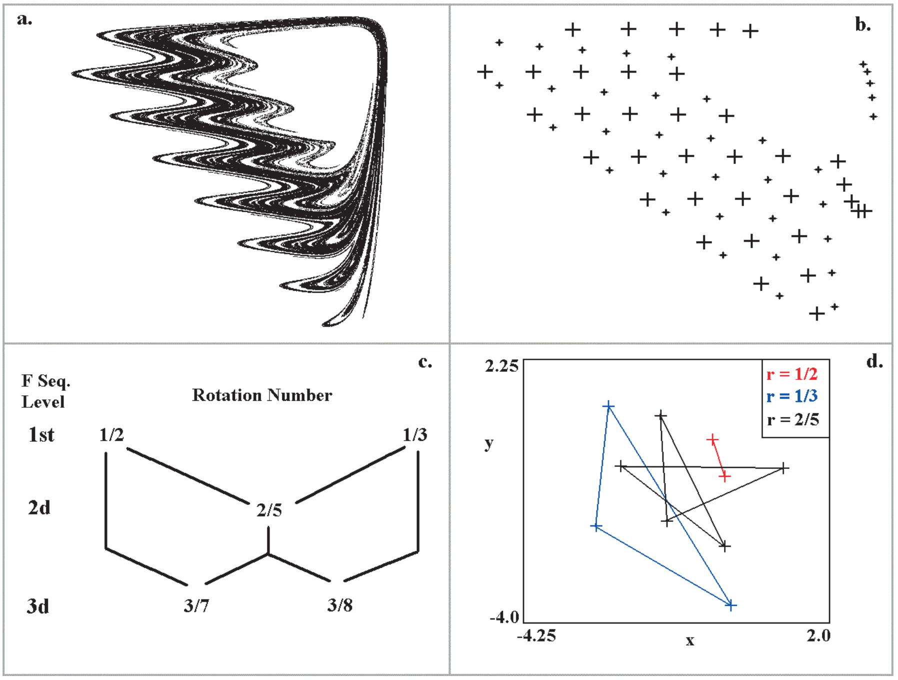


Figure 1. Periodicity in chaos. **a.** Chaotic attractor in a predator-prey model subject to seasonal forcing. The attractor is viewed stroboscopically by sampling trajectories at yearly intervals, which are equal to the period of forcing. Axes are logarithms of the numbers of predators and prey. **b.** Non-stable cycles embedded within the chaotic attractor corresponding to rotation numbers, $r$ = 1/7, …, 1/11, *i.e.*, to periods of 7, …, 11, with one peak per cycle). The cycles come in pairs, reflective of the bifurcations that produce them. All are saddles, a condition necessitated by the transitive character of chaotic sets *[Devaney, 1987]*. **c.** Farey sequence construction showing the first three levels between 1/2 and 1/3. In the limit of an infinite number of levels, one obtains all of the intervening rational numbers. **d.** Cycles corresponding to rotation numbers, $r$ = 1/2, 1/3 and 2/5 for the predator-prey model. In **a.** and **b.**, the time scale of the unforced system is $2\pi$ simulated years; in **d.**, $(2\pi/3.1)$y. For additional details, see King and Schaffer *[1999]*.

With regard to anthropogenic warming, it follows that there may be concentrations of greenhouse gasses sufficient to obliterate intrinsic variability, at which point, the climate will simply track external drivers.[4] Whether or not the world is close to such a point is unknown. In the first place, so far as I am aware, the question has never been asked. Second, there is continuing disagreement as to the impact of atmospheric carbon *[Caillon et al. 2003; Bellamy and Barrett, 2007; Douglass et al, 2007; Ahn and Brook, 2008]*. The consensus view, as expressed by Wood is that anthropogenic forcing will override natural variability; its antithesis, that natural variability is

[4] This is in contrast to the conventional assertion that AGW implies increased variability.

"unstoppable" *[Singery and Avery, 2007]* and that correlated increases in $[CO_2]_{atm}$ and temperature are coincidental.

**Spontaneous Climate Change.** Relevant to these alternatives is a recent paper by Tsonis *et al. [2007]* on climate change consequent to varying levels of coupling and synchronization among semi-isolated oscillators (PDO, NAO, ENSO, NPO). When synchronization was followed by continuing increases in coupling strength, a new climate state emerged. Such events, the authors argue, are observable in both historical data and *in simulo*, and, in the models, both with and without increasing concentrations of greenhouse gases. They write:

> "The fact that this mechanism is present in the control run [pre-industrial conditions] will indicate that the shifts are not caused by some kind of bifurcation (which will require [changing] external influences) but rather it is an intrinsic property of the climate system."

Referring to post-1970's climatic warming, they further suggest that

> "The standard explanation for the post 1970s warming is that the radiative effect of greenhouse gases overcame shortwave reflection effects due to aerosols … However, comparison of the [simulated] 2035 and the 1910s event [historical data] in the observations … suggests an alternative hypothesis, namely that the climate shifted after the 1970s event to a different state … , which may be superimposed on an anthropogenic warming trend."

Tsonis e*t al*. (see also, *[Tsonis et al. 2008]*) use techniques from network theory *[Strogetz, 2001]* in their analysis and so-called synchronized chaos *[Boccaletti et al., 2002]* as context for discussion. But this is icing on the cake. To evolve chaotically is to dance the dance the cycles. By way of contrast, the overwhelming bulk of the climate literature discusses internal variability either in terms of proximate mechanisms, such as MOC, or as possibly amplified responses to external forcings, for example, volcanic irruptions and variations in solar *[Solanki, 2002; Scafetta and West, 2006]* and cosmic radiation *[Svensmark and Friss-Christensen, 1997]*. The former view neglects the origins of mechanistic variability, while the latter assumes one-to-one correspondence between periodic driving and climatic response.

**Pitfalls of Linear Thinking.** Force fitting empirical observations to preconception is risky. Here are two additional properties of nonlinear systems that get swept under the rug by linear thinking.

1. **Fluctuating Parameterizations.** Takens *[1981]* (see also, Sauer *et al. [1991]*) proved that the observables of non-equilibrium systems generically manifest the same qualitative dynamics as the system itself. By "observables," I mean the state variables and quantities that can be computed therefrom; by "generically,"

that the probability of any particular observable's manifesting said dynamics is one, even though there can be many exceptions. An example is climate sensitivity, which may be defined as the change in equilibrium temperature consequent to doubling $[CO_2]_{atm}$ *[Roe and Baker, 2007]*. Sensitivity is a far more complex function of system state than the quantities conventionally used for reconstructing phase space dynamics. Nonetheless, if climate evolves in finite-dimensional subspaces – more precisely, on compact manifolds – Takens' result applies. In short, if the climate is chaotic, so too is sensitivity, and with probability one.

2. **Clouded Causality.** The conventional view is that increasing greenhouse gas concentrations induce higher temperatures. But we now know *[Caillon et al. 2003]* that increasing temperatures can precede a rise in greenhouse gases. Of course, one can account for this with *ad hoc* mechanistic hypotheses. But this misses the broader point, which is that relationships among variables in chaotic systems can change as the system evolves – see, for example, King and Schaffer *[2001]*, their Figure 10c.

**Nonlinear Resonance**. The response of linear oscillators to periodic forcing is straightforward. If the forcing frequency is rationally related to the intrinsic frequency of the oscillator, resonance ensues. Otherwise, the result is quasiperiodicity. In nonlinear systems, things are more complicated. Often one observes complex parametric dependencies *[Schaffer and Bronnikova, 2007]*. Regarding these dependencies, one may observe the following:

1. The parameter space is typically divided into regions corresponding to the existence of cycles of different topology *[Arnol'd, 1983]*. Within these regions, cycles may be stable or unstable. Often the regions overlap – hence the coexisting cycles in Figure 1b.

2. Some of these regions correspond to cycles that are *modulations* of the forcing function. For example, if the driver is seasonality, such oscillations manifest yearly peaks of varying amplitude that repeat every $T > 1$years.

3. Other resonances are termed *subharmonic*, inasmuch as they manifest a single peak per cycle with a period greater than that of the forcing function. Typically, there is a resonance of minimum period, $T_0$, determined by the natural time scale of the unforced system. In the case of annual forcing, one has additional resonances of period $T_0 + 1$, $T_0 + 2$, …, which is the situation shown in Figure 1b.

4. More complex resonances (ultraharmonics) also obtain. Periodic orbits can be characterized by their rotation numbers, $r = p/q$, where $q$ is the number of points on the cycle (when viewed in section), and $p$ is the number of rotations about the circle necessary to visit all of them. The available cycles can be identified via construction of the so-called Farey sequence (Figure 1c), which is a way of

constructing the rational numbers. Specifically (Figure 1d), between cycles with $r_1 = p_1/q_1$ and $r_2 = p_2/q_2$, one expects a third cycle, with $r_3 = (p_1 + p_2)/(q_1 + q_2)$.

Relevant to points #3 and #4 is the suggestion *[Huybers and Wunsch, 2005]* that of the three Milankovitch forcing frequencies, only obliquity ($T \approx$ 40Ky) is statistically significant. According to this "ice sheets terminated every second or third obliquity cycle at times of high obliquity … ." This suggests shuttling between the $r$ =1/2 and $r$ = 1/3 cycles and possibly the $r$ =2/5 cycle that occurs between them.

A second straw in the wind is the observation *[Huybers and Curry, 2006]* of spectral power law scaling over annual, decadal and Milankovitch frequencies, but with two different exponents – the break coming at around frequencies of about 1/100 $y^{-1}$. This suggests, as one might predict, that decadal oscillations are subharmonic resonances of the solar or yearly cycles.

Finally, there is the suggestion *[Braun et al. 2005]* that so-called "Dansgaard-Oeschger" (1470 y) events are resonant responses to solar cycles of -87 and -210 years. The authors note that 210 divides into 1470 seven times, suggesting an $r$ =1/7 resonance.

**Implications for Policy.** Proposed solutions to the "climate crisis" balance short-term costs and long-term benefits. Setting aside disagreements *[Byatt et al. 2006; Stern et al. 2006; Tol and Yohe, 2006]* regarding particulars, the considerations reviewed here suggest that anticipated long-term benefits should be discounted by uncertainty as to just how the climate will actually evolve. That the current stabilization of global temperatures was unforeseen twenty years ago should give pause to those who would impose draconian policies on today's population in the name of saving the planet. Things could easily go the other way, with a new Little Ice Age or something worse in the near- to moderate-term future. Should ice, not fire, be in our future, carbon abatement policies will exacerbate the ensuing (non-mathematical) chaos. As for physicians, the watchword for policy makers should be "First do no harm." Or, to put it another way, the precautionary principle cuts both ways.

**Darwin and the Physicists.** I began this review by quoting from one of the papers that led to the discovery of evolution. Let me close by returning to that subject. In his testimony to the U.S. Senate Energy and Public Works Committee, Professor William Happer *[2009]* recalled Lord Kelvin's "back of the envelope" calculation *[Thomson, 1866]* that seemingly invalidated the Lyellian postulate of near-limitless time during which natural selection could mould one species into another, bit by imperceptible bit. As Happer points out, Kelvin was wrong; his computations, a spectacular example of model failure in physics. But there is more to the story. Kelvin's limit to the age of the earth, which was grudgingly accepted by contemporary geologists *[Geikie, 1895]*, along with some trenchant observations on variability by Fleeming Jenkin *[1867]*, went unanswered for the next forty years *[Eiseley, 1961; Bowler, 1983]*. What kept the idea of evolution alive, what led to its near-universal acceptance, was the continuing accumulation of evidence from other sources, anatomy, development, biogeography, systematics and especially paleontology, evidence for which the most economical explanation was descent with modification. As climate modelers refine their computer programs, one can only hope that the worth of these productions will be judged by their ability to iterate to the data. The Little Ice Age was real, as changing isotopic ratios in Viking bones buried in Greenland *[Arneborg et al. 1999]* so eloquently attest.

**Note Added in Press.** Swanson and Tsonis *[Swanson, K. L., and A. A. Tsonis. 2009. Has the climate recently shifted? Geophys. Res. Lett. doi:10.1029/2008GL037022, in press.]* now conclude that the earth's climate did, in fact, shift in 2001-2002. As in their previous paper *[Tsonis et al. 2007]*, they associate climate change with the synchronization of chaotic oscillators. The resonance is, of course, non-stable, being one of the saddle cycles about which the climatic attractor is organized. Hence, the transitory nature of the synchronization and subsequent shift to a new state.

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