The Misuse of Mathematics

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Abstract The computer revolution has begotten new branches of mathematics, eg, chaos theory and fractal geometry and their offspring, agent based modeling and complex dynamical systems. These new methods have extended and changed our understanding of the complex systems in which we live. But math models and computer simulations are frequently misunderstood, or intentionally misrepresented, with disastrous results. In this essay, we recall the concept of structural stability from dynamical systems theory, and its role in the interpretation of modeling.

1 Introduction

We begin by recalling the history of the modeling of complex dynamical systems. The first step, following the computer revolution, was the development of system dynamics by Jay Forrester in the 1960s. Many system dynamics models behaved chaotically, as we would now expect, but at the time this irregular behavior was considered misbehavior, and the supposedly faulty models were ignored. The advent of chaos theory in the 1970s breathed new life into system dynamics. In chaos theory, the long-term behavior of a dynamical system is described by attractors, of which there are three types: static, periodic, and chaotic. And thus, the misbehavior of a system dynamics model became the chaotic behavior of a complex dynamical system, modeled by a chaotic attractor.

But the chaotic attractors of a complex dynamical system suffer from sensitivity to initial conditions (the butterfly effect) and thus cannot be used for quantitative prediction. The modeling activity is nevertheless crucial to the hermeneutical circle that drives the advance of science. The qualitative behavior of a model provides a cognitive strategy for understanding the behavior of a complex dynamical system. But even the qualitative behavior of a model cannot be trusted as an indicator for

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the natural system being modeled, due to a problem called structural instability, as we describe in this article.

In short, mathematical modeling is valuable for comprehension, but not for prediction. Ignoring this simple fact is the cause of much of the misuse of mathematics in contemporary policy making, including the examples described below.

2 Useless arithmetic

In *Useless Arithmetic: Why Environmental Scientists Can't Predict the Future*, environmental scientists Orrin Pilkey and Linda Pilkey-Jarvis present several cases of the misuse of mathematical modeling, including the collapse of Atlantic cod stocks, prediction of stock prices, body counts during the Vietnam war, the safety of nuclear waste storage at Yucca Mountain in Nevada, the rise of sea levels due to global climate warming, shoreline erosion, toxicity of abandoned open-pit mines, and the spread of non-indigenous plants. The main modeling strategy in these case studies is that of complex dynamical systems. This type of model, for reasons spelled out in detail below, cannot be relied upon for prediction. Nevertheless, policy makers with their own agendas may fool people (and themselves) into accepting risky policies by misrepresenting simulated data as prediction. This is what the Pilkeys mean by useless arithmetic. But it is worse than useless, it is dangerous.

In their first case study of useless arithmetic, the collapse of the the North Atlantic Cod stocks, the results of simulations were misused by the fishing industry and the Canadian government to sell the public on a fishing policy that essentially killed the cod fisheries, and the cod fishing industry. The mathematical models used were simple dynamical systems derived from the population dynamics of a single species, the Atlantic cod. [8; p. 10] Interacting populations in the ecosystem were ignored. The quantitative predictions of these models overestimated the safe catch, and the collapse of the Grand Banks cod fishery in 1992 was the result.

Even if other factors were included in a complex dynamical model, chaos theory implies that even qualitative predictions are unreliable. Chaotic attractors, fractal boundaries, and catastrophic bifurcations all mitigate against credible forecasts. As argued in this essay, it is not practical to establish that a model is structurally stable, that is, qualitatively reliable.

For example, consider the classic model for two interacting species, the Volterra-Lotka model. This was first proposed independently by the American mathematical biologist Alfred Lotka (1880-1949) in 1925 and by the Italian physicist Vito Volterra (1860-1940) in 1926 to model predator-prey dynamics. [5] This model displays periodic behavior no matter what the initial conditions are. But a small perturbation in the model can produce behavior in which all trajectories spiral to the origin, that is, the oscillations die out. Despite this structural instability, the model is pedagogically useful in teaching basic principles of population dynamics.

3 Structural stability

We may use this example to introduce the concept of structural stability. (See [2] for more on this.) The dynamics of the predator-prey models are shown in Figure 1. These show the number of individuals in the two populations graphically: the number of prey (small fish) on the horizontal axis, and predators (big fish) on the vertical axis. The trajectories circling counterclockwise reveal this periodic cycle:

In Figure 1 (left panel), the basic cycle is shown. Beginning at top dead center, predators are at maximum population, prey are declining as so may predators are eating them. At the left extreme, pray are at minimum, while predators are decreasing as there is not enough to prey for them to eat. At the bottom of the cycle, predators are at minimum strength, so prey are again on the increase. At the left extreme, prey at maximum strength, so predators have plenty to eat and are on the increase.

The right panel of Figure 1 shows the cycles for several different starting conditions. This configuration of concentric cycles is called a *center* in dynamical systems jargon. It is an example of structural instability, as shown in Figure 2.

If small vectors are added to the dynamic, each one pointing radially toward the center, we obtain a new system in which the trajectories spiral in to the red point, which is a point attractor, as shown in Figure 2 (left panel).

This system and the preceding one (with concentric cycles) are not qualitatively equivalent: one has periodic behavior, the other has an attractive equilibrium point. Figure 2 (right panel) shows another portrait obtained from the Volterra-Lotka

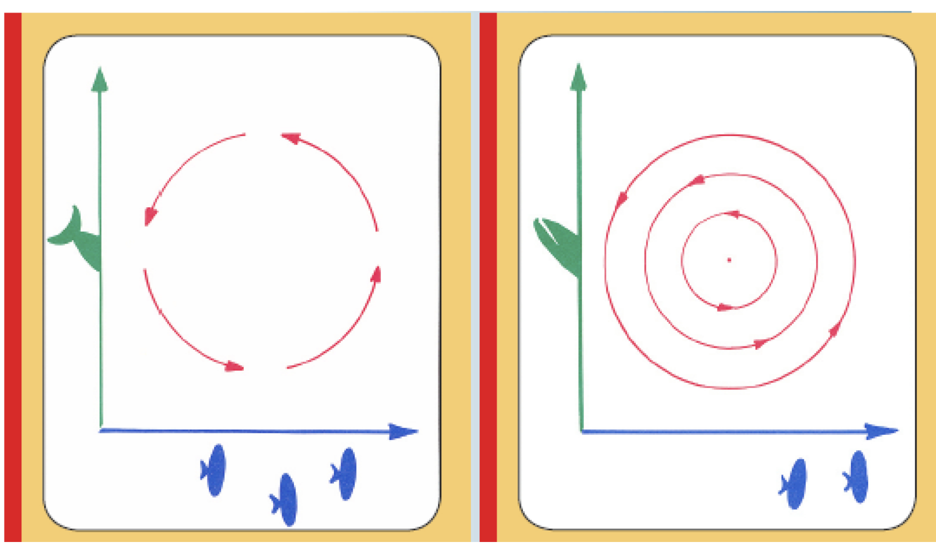

Fig. 1 Phase portrait for the Volterra-Lotka vectorfield. [1; Part One]

model by a small perturbation. This one has a single periodic cycle, shown in red, into which nearby trajectories spiral.

Dynamical systems theory has shown that these latter two portraits are *structurally stable*: given a small perturbation, the portraits are not qualitatively changed. In fact, two-dimensional systems have a special situation in dynamical systems theory: any such system may be perturbed into a structurally stable system. This was proved by the Brazilian mathematician, Mauricio Peixoto, in 1958. [1; Part Three] But if we add a third population, we encounter a serious problem: in dimensions three or more, there are large sets of dynamical systems which are robustly unstable. This aspect of chaos theory presents an insurmountable problem for the interpretation of dynamical models.

4 The climate skeptics

Al Gore and the Intergovernmental Panel on Climate Change (IPCC) shared a Nobel prize in 2007 for their work on climate prediction. Global climate warming has been, is, and always will be, controversial. Skeptics have called it the greatest scientific hoax of all time, and the IPCC has been accused of major deception. Millions of people have seen Al Gore climbing a ladder to show the predicted rise in sea level. James Lovelock, the Gaia Hypothesis guru, expects a rise of 200 feet. [7] Meanwhile, the IPCC expects 2 feet. Many climate models have been extensively studied, from the simple two-component *Daisyworld model* of James Lovelock, to massively complex models including most of the known factors. In this section we will briefly summarize a few of the skeptical accusations.

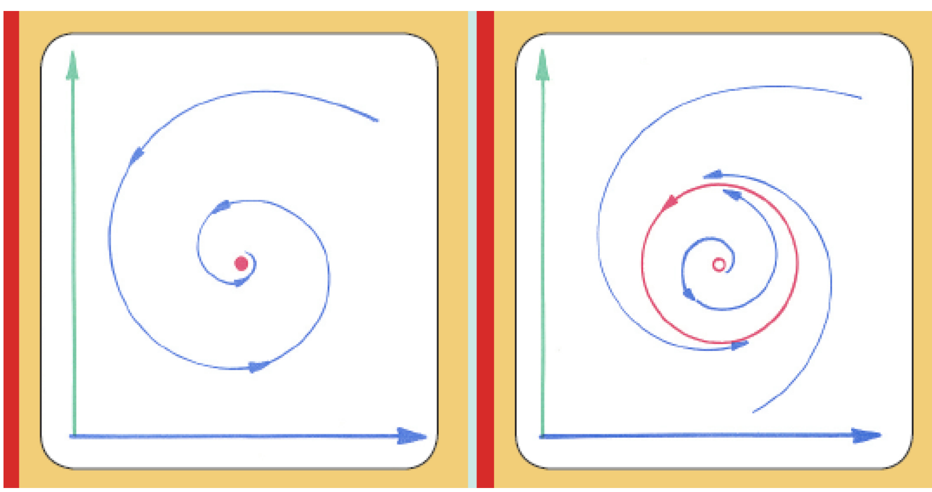

Fig. 2 Phase portrait for the perturbed vectorfield. [1; Part One]

Bjorn Lomborg, in the *Wall Street Journal* of November 2, 2006, criticizes a 700-page report by Nicholas Stern and the U.K. government, for using faulty reasoning and data in estimating the cost of excess atmospheric carbon.

Mario Lewis, Jr., in the *Competitive Enterprise Institute* website and on C-Span on March 16, 2007, takes Al Gore to task for his film and book, *An Inconvenient Truth*. He believes that most of Gore's claims regarding climate science and climate policy are either one sided, misleading, exaggerated, speculative, or wrong.

Freeman Dyson, Nobel laureate, at the University of Michigan, Winter 2005, called global warming the most notorious dogma of modern science. In an interview in the *TCS Daily* of April 10, 2007, he explained,:

> Concerning the climate models, I know enough of the details to be sure that they are unreliable. They are full of fudge factors that are fitted to the existing climate, so the models more or less agree with the observed data. But there is no reason to believe that the same fudge factors would give the right behavior in a world with different chemistry, for example in a world with increased CO2 in the atmosphere.

Stewart Dimmock of the New Party in the U. K. sued the government for distributing the Gore film, citing nine inaccuracies. Most damaging, in my opinion, is this one:

> The film suggests that evidence from ice cores proves that rising CO2 causes temperature increases over 650,000 years. The Court found that the film was misleading: over that period the rises in CO2 lagged behind the temperature rises by 800-2000 years.

While global climate warming may yet be catastrophic, this experimental observation suggests that perhaps human burning of fossil fuel may not be causative. However, atmospheric methane rise does foreshadow warming.

John Coleman, founder of the Weather Channel in the US, as reported in the *London Telegraph* on September 11, 2007, wrote in ICECAP,

> It is the greatest scam in history. I am amazed, appalled and highly offended by it. Global Warming; It is a SCAM.

John R. Christy, a member of the IPCC, writes in the *Wall Street Journal* of November 1, 2007:

> I'm sure the majority (but not all) of my IPCC colleagues cringe when I say this, but I see neither the developing catastrophe nor the smoking gun proving that human activity is to blame for most of the warming we see. Rather, I see a reliance on climate models (useful but never "proof") and the coincidence that changes in carbon dioxide and global temperatures have loose similarity over time. ... It is my turn to cringe when I hear overstated-confidence from those who describe the projected evolution of global weather patterns over the next 100 years, especially when I consider how difficult it is to accurately predict that system's behavior over the next five days.

The Committee on Environment and Public Works of the U.S. Senate, in a minority report on December 20, 2007, reports that 400 prominant scientists from 24 countries dispute man-made global warming.

Ferenc Miskolczi, an atmospheric physicist formerly of NASA's Langley Research Center, reported at the International Climate Change Conference of March, 2008, that the dynamical model usually used for the unlimited greenhouse effect was missing a term. The corrected equations predict an upper limit to the greenhouse effect.

Well, this is enough to give some credibility to the climate skeptics, who had their own conference in New York in March 4, 2008. For links to online sources for all these and more see [3].

In summary, we have this conundrum: yes, climate is warming, as it periodically does. Even if this warming tops all prior warmings due to human-produced greenhouse gas emissions, we still can not predict, on the basis of a mathematical model, whether the climate will stay warm, or rather, cool down again in a new ice age, as it has eight times in the past 650,000 years.

5 Daisyworld

We now turn to a climate model that epitomizes the pedagogical value of modeling, despite being radically simplistic. This is the Daisyworld model of James Lovelock. After the publication of his Gaia Hypothesis in 1979 [6] earned him the scorn of the earth science community, Lovelock created the Daisyworld model to help people understand how Earth's biosphere could regulate its own climate. [9] Here is the idea.

Daisyworld is a fictitious planet having only two living species: white daisies and black daisies. The white daisies make the planet cooler by reflecting visible sunlight back into space, but they prefer (that is, grow faster) in a warmer climate. Meanwhile, the black daises make the planet warmer by absorbing the suns visible rays, and reradiating the energy as infrared, but they prefer a cooler climate. The daisyworld planet, partly white daisies, partly black daisies, and partly bare dirt, acts as a thermostat. A stable temperature is maintained, even when the brightness of the sun (solar luminosity) increases, as shown in Figure 3.

The equations given by Watson and Lovelock describe a structurally stable system in two dimensions. However, if more species of daisies are added, stability may become problematical.[41]

6 Conclusion

The interpretation of nearly all dynamical models has to be carried out cautiously due to the likelihood of structural instability. This means that the behavior of the model might change drastically due to a small change in the model. It would be nice if a given model could be simply tested for structural stability, but there is no such test. Thus, the goal of modeling is pedagogic, not predictive in the long term. For example, global climate models cannot tell us how much sea level will rise, nor how long a given rise will take, and not even, if the current rise will be followed by an ice age, or a permanent interglacial climate. So, better to be safe than sorry!

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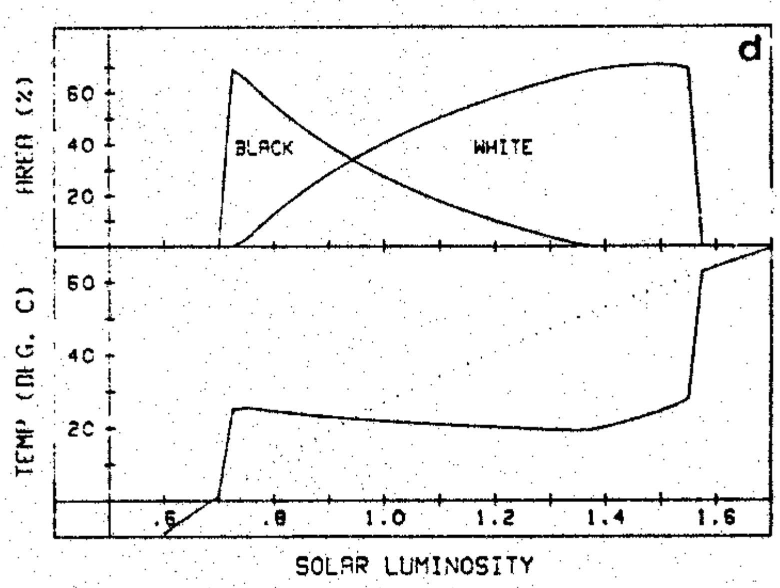

Fig. 3 Thermostatic behavior of the Daisyworld model. [9]

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