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On a possibility of estimating the feedback sign of the Earth climate system

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Abstract. The growth rate of the second moment for the time series' increment as a function of the increment range can be used for estimating the sign of feedback of the underlying physical system. The influence of the periodic nature of the time series to the growth rate of its structure function is considered. The approach is used to describe the variability of the time series of the global average outgoing long wave radiation (OLR). It is shown that the series' annual cycle plays a crucial role in preventing the growth of the variance of the time series' increments and leads to its nearly stationary long-range behaviour. The analysis of the OLR time series indicates that a negative feedback should dominate in the earth climate system. The example is believed to be useful for better understanding of the influence of the increasing concentration of CO_2 in the Earth's atmosphere.

Key words: Earth climate system, feedback sign, time series analysis.

1. INTRODUCTION

One of the largest challenges in the understanding of the behaviour of the Earth's climate variations consists in adequate estimation of the influence of feedbacks in the climate system. The sign of the feedbacks is the key element of the system. Currently, the feedback, connected to the increase of CO_2 concentration in the earth atmosphere, presents the main scientific interest [1].

A widely accepted understanding ([2], chapter 9) is that the equation for the average radiation budget B at the top of the atmosphere

$$B = I(1 - a) - F = 0, \tag{1}$$

where I is the incident solar flux, a is the albedo of the system earth–atmosphere, and F is the outgoing long wave radiation flux, can be used to estimate various feedbacks in the earth climate system.

Since Eq. (1) does not take into account the temporal variability, its use for estimation of the long-term behaviour of B is questionable. In reality, due to the eccentricity of the Earth's orbit, the actual situation depends on time and all processes in the earth's climate system are under a strong influence of the annual cycle of the radiation budget of the Earth.

The annual variation in the global net radiation budget was considered by Simpson [3]. Vonderhaar and Suomi [4] indicated the possibility to obtain the annual variation from limited sets of former satellite data. The first empirical determination of the annual variation was carried out in [5], based on 29 months of satellite data. The available datasets have been remarkably increased now and the properties of the radiation budget have been established with a high accuracy.

The sign of the overall feedback of the global climate system is important to comprehend the climate system variability. Hansen et al. [6] define climate feedbacks as *internal reactions of the climate system to (natural or anthropogenic) climate change*. This statement may cause some confusion. The state of the climate system changes continuously due to the periodic solar forcing. This means that a separation of the system's change due to the outside forcing from the feedback is methodically difficult. The fact that climate change is considered to happen over a remarkably longer time period than the annual cycle does not help here. No-one has ever succeeded in showing that the main climate variables (such as the surface air temperature) present stationary time series. *Vice versa*, expectations about non-stationarity can be found quite often [7]. This indicates that the detection of the actual climate change by means of traditional thinking, based on the expected stationarity of a stable climate, may be too simplistic.

It is evident that the separation of the changes due to the annual cycle in the solar forcing from those occurring due to the feedback, caused by, e.g. the reaction to the CO_2 forcing, is impossible by means of any data analysis. Thus, we need to estimate the feedback together with the customary forcing cycle and to use indirect ways for that. The processes, called fractional Brownian motions (fBm), enable us to do that on the basis of the fitted Hurst exponent H ($0 < H < 1$) over the time interval of interest [8]. The fBm has an important property, namely, a significant long-range correlation between its consecutive non-overlapping increments. Similar correlations between the series increments can be used for the estimation of the sign of the feedback.

Geophysical time series often behave scale-invariant over some time interval [9]. Thus, one can interpret the exponent H as the indicator of the sign of the feedback over that time scale. An analysis of several surface air temperature time series in [10] showed that the estimates of H and correlations between the consecutive increments led to the same result (namely, to an anti-persistent nature of the underlying system) over a sufficiently long time interval.

In the present study the global mean outgoing long wave radiation series are used to explain the problems, occurring in the attempts of estimating the nature of the overall feedback of the Earth's climate system to the existing forcing. The

OLR describes the response to the forcing. Its variability can be used for estimating whether the amplitude of the main forcing is amplified or suppressed through the feedback loops. This is an attempt to explain how the annual cycle plays a crucial role in causing the long-term anti-persistence in the globally averaged case. It is shown that the annual cycle bounds the growth of the time series' increments variance while the increment range increases, because, energetically, the annual amplitude of the increments is larger than the standard deviation of its anomalies.

2. THE TEMPORAL VARIABILITY OF THE OLR

The temporal variability of the climate system can be analysed on the basis of the flux density F of the outgoing long wave radiation. It represents directly the response of the climate system to the changes in solar forcing. The result can be characterized by means of the effective temperature T_e, where $F = \varepsilon\sigma T_e^4$, ε is the emissivity of the radiating object, and σ is the Stefan–Boltzmann constant.

Satellite measurements of OLR by means of the advanced very high resolution radiometer (AVHRR) have been carried out since 1974. The results have been archived and are available on-line (dss.ucar.edu). In the present study the 5-day averaged global values (pentads) from the time interval 1974–1999 are used. The period from April 16, 1974 to December 31, 1978 in this record contains interpolated data, because no actual measurement results are available for that period.

A 10-year example of OLR and T_e series is shown in Fig. assuming $\varepsilon = 1$. Both series have a periodic nature. The cycle can be approximated as follows:

$$X(t) = A_0 + A_1 \cos\left(\frac{2\pi t}{365} + \phi_1\right), \tag{2}$$

where $A_0 = 231.2$, $A_1 = -4.55$ and ϕ_1 =- 0.210 for the OLR record and $A_0 = 252.69$, $A_2 = -1.24$ and $\phi_2 = -0.211$ for T_e. The use of more harmonics will lead to a better fit but is not necessary for the current task, because an adequate description of the periodic component with the largest amplitude is sufficient to describe the main idea of the paper.

We can write the time series of quantities F and T_e as follows:

$$\begin{aligned} F(t) &= F_a(t) + \epsilon_1(t), \\ T_e(t) &= T_{ea}(t) + \epsilon_2(t), \end{aligned} \tag{3}$$

where F_a and T_{ea} are determined by Eq. (2) with appropriate constants A_j, and $\epsilon_1(t)$ and $\epsilon_2(t)$ are the corresponding anomaly series (i.e. the residuals in respect to the mean annual cycle, respectively). One can expect that the influence of growing CO_2 concentration is hidden mainly in the residuals.

In order to get an idea about the variability range, the annual deviations of F from the sample mean value 231.2 Wm^{-2} are shown in Fig. 1b.

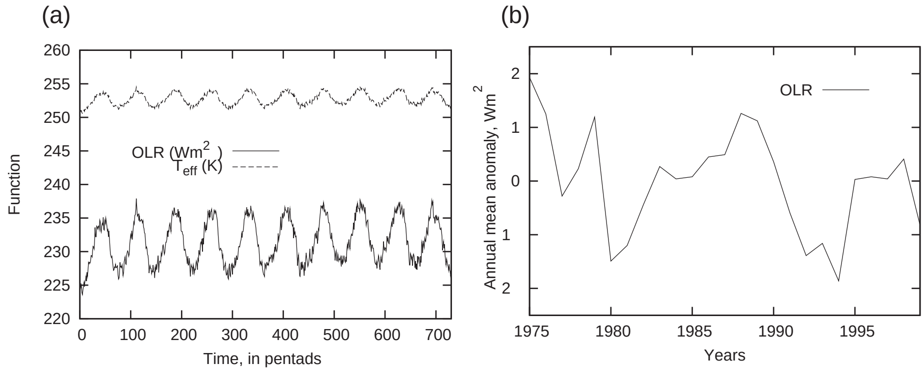

Fig. 1. (a) Series of 5-day mean values for the OLR and T_e during 1981–1990; (b) annual OLR deviations from the mean 231.2 Wm^{-2} during 1975–1999; the value for 1978 is interpolated.

3. QUANTIFYING NON-STATIONARITY

A particular value of the time series $X(t)$ at the time instant t can be presented by

$$X(t) = \sum_{i=0}^{\infty} x(t-i), \tag{4}$$

where $x(t) = X(t) - X(t-1)$ is the corresponding increment during the time step t.

The temporal variability of the non-stationary series $X(t)$ can be investigated on the basis of the growth rate of variance for the increment

$$X(t+\tau) - X(t) = x(t+1) + \ldots + x(t+\tau), \tag{5}$$

where $t = 0, 1, \ldots, T-\tau$, as a function of τ. The properties of this function are determined by the increments $x(t)$.

The necessary function, representing this growth rate (structure function or variogram) can be written as

$$\begin{aligned} D(\tau) &= \frac{1}{T-\tau} \sum_{i=1}^{T-\tau} (X(i+\tau) - X(i))^2 \\ &= \frac{1}{T-\tau} \sum_{i=1}^{T-\tau} (x_{i+1} + \ldots + x_{i+\tau})^2 \\ &= \tau[C(0) + 2\sum_{i=1}^{\tau-1} (1 - i/\tau) C(i)], \end{aligned} \tag{6}$$

where $C(i)$ stands for the auto-covariation of the increments $x(t)$ at the lag i.

Expression (6) shows that the growth rate for $D(\tau)$ depends on the correlations between the increments (over the range $1\ldots(\tau-1)$). It is customary to estimate it by means of a special Hurst exponent H [8] defined by the relation $D(\tau)\propto\tau^{2H}$, where $0<H<1$. The value of H determines some important classes among the non-stationary series.

1. If $C(i)=0$ for all $i>0$, then Eq. (6) shows that the growth rate for $D(\tau)$ is proportional to τ, and consequently $H=0.5$. The process with independent identically distributed increments (random walk) is the best known example among those having $H=0.5$.
2. If positive correlations are dominating in the term $\sum_{i=1}^{\tau-1}(1-i/\tau)C(i)$, the function $D(\tau)$ has a faster growth rate than in the previous case. Positively correlated increments tend to have the same sign, so the process tends to increase in the future if it has had an increasing tendency in the past. And, *vice versa*, it has a tendency to decrease in the future if it has had a decreasing tendency in the past. Such a feature is called *persistence* (P) [8]. Physically, a persistent system is going to increase the deviation, thus a positive feedback generally dominates in the system that governs the time series. It is convenient to consider the processes, where the growth rate is proportional to τ^{2H} and where $0.5<H<1$ is constant, over some (limited or infinite) interval of τ.
3. If over some interval $\tau_0<\tau<\tau_1$ the function $D(\tau)$ is growing more slowly than in case (1) so that $D(\tau)\propto\tau^{2H}$, where $0<H<0.5$, negative correlations dominate in the system. Being negatively correlated, the increments tend to have opposite signs, so that the process has a tendency to decrease in the future if it has had an increasing tendency in the past and *vice versa*. The feature is called *anti-persistence* (AP). It expresses a tendency of the values of increments to compensate each other to prevent for the process from blowing up too fast. Such a system tends to eliminate deviations showing a negative feedback in aggregate.
4. If $H=0$, the necessary condition for stationarity of $X(t)$ is satisfied [11].

To get the actual estimate, the exponent H is fitted to the change of $D(\tau)$ over some finite interval for the time lag τ ($\tau_0\le\tau\le\tau_1$) by means of the linear equation

$$\log D(\tau)=2H\log\tau+K, \tag{7}$$

where K is a constant.

The latter has no significance in the current study.

The growth rate for $D(\tau)$ can be calculated for every sample of the time series. The rate can be quantified by H over the periods where it is approximately linear. The results can be characterized by means of the feature of persistency (AP or P) in the same way as for theoretical models. The scheme enables us to estimate the sign of the feedback on the basis of the assigned persistency property (as a function of the time scale). A similar estimation has been carried out for various surface air temperature series in [10].

Studies of the variability of the global radiation budget have shown that the annual cycle dominates in its variability. This means that the same cycle should have a significant influence on any global variable describing the state of the Earth's climate system. Three examples about the growth rate of $D(\tau)$ for simple theoretical series to explain the influence of combining an annual cycle with a certain noise series will be presented in the form

$$X_1(t) = F_a(t) + z(t), \qquad t = 1, \ldots, 262144, \tag{8}$$

where F_a stands for the periodic (see Eqs. (2) and (3)) part and z is specified separately in every case.

A periodic series cannot be a mono-scaling one in terms of H. A scale break should occur approximately if τ grows larger than a half of the period. Our main interest is whether the condition $H > 0.5$ may be satisfied for these combinations where τ becomes sufficiently large. It is easy to show that the growth rate for F_a shows AP only if τ grows for a sufficiently long period in comparison with the main period.

Figure 2a shows the growth rate of $D(\tau)$ for three combinations involving F_a. Let us consider first the case $z(t) = 0.0004t$. It represents the situation where a linear trend takes the lead over a long-range stationarity of a periodic function for large values of τ.

One can imagine it considering a growing influence of the CO_2 forcing (the second term) in comparison with a variable, containing a determined annual cycle with $A_1 = 4.55$. The trend may be caused by the influence of growing CO_2 concentration in the earth's atmosphere. The numerical value of the slope (0.0004) appears to be quite arbitrary. It is not fitted for the radiative effect of the the daily increase of the CO_2 concentration. No such fitting is necessary, because the current example presents a sum of two independent processes. Such a situation is not likely in the Earth's atmosphere.

The growth rate of $D(\tau)$ in this example explains the scale dependence of H, that is, the case when the feedback of a real system can also be scale dependent.

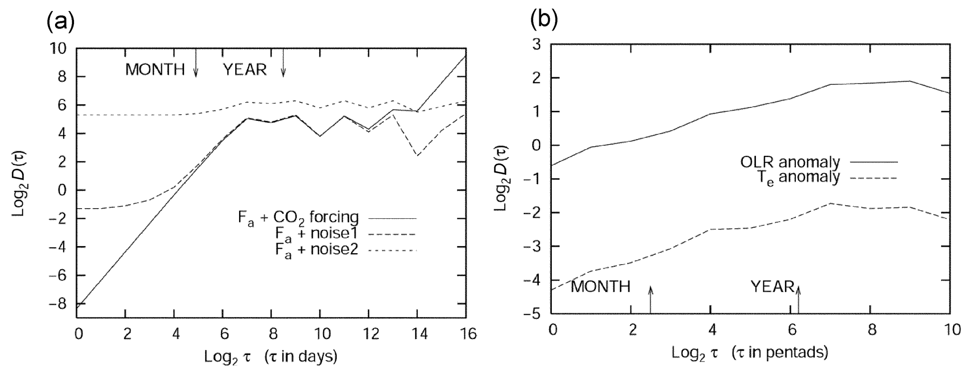

Fig. 2. The growth rate of $D(\tau)$: (a) for F_a (OLR annual cycle) plus some function; (b) for the deviations of OLR and T_e from their mean annual cycle.

The function $D(\tau)$ grows rapidly ($H \cong 1$) for $1 < \tau < 128$ days due to one phase of the annual course. The scale break takes place for a certain τ, because the covariations between the increments start turning negative if the increment range exceeds a quarter of the main period (see Eq. (6)). For $128 < \tau < 16384$, the function $D(\tau)$ keeps undulating around the reached value. The undulation is caused by the use of a sparse grid; however, a more accurate course is not important for the current study. The trend is still low to dominate in the behaviour of $D(\tau)$ during that τ interval ($128 < \tau < 16384$), and the approximate value for H would be zero.

The value of τ, at which the function $D(\tau)$ starts another rapid growth, enables us to determine how "high" should reach the trend (in comparison with the annual amplitude of $F_a(t)$) in order to evoke a scale break (in terms of H). If $t = 16384$ the "height" of the trend has reached 6.55$> 4.55 = |A_1|$ and for even larger values of τ we have $H \cong 1$ again. The trend obviously remains dominating over even longer scales.

This example explains two important issues connected to the quantification of non-stationarity in time series of the type "determined cycle + trend". There are two essential scale breaks: the half-length of the cycle and the interval after which the trend "height" becomes equal to the amplitude of the cycle. The first break leads from the initial rapid growth based non-stationary regime to a (quasi)stationary one due to the stabilizing influence of the annual cycle. The second scale break leads from the (quasi)stationarity back to a "new" non-stationary behaviour due to the fact that the "height" of the trend becomes larger than the annual amplitude in Eq. (8) and the trend term starts dominating the variance growth. Thus, this example reveals that a large annual amplitude presents an important threshold in estimates of the persistence in the series with various outer forcing.

Two other examples represented in Fig. 2 are of the type F_a + white noise. Their difference is in the standard deviation: $\sigma_1 = 0.1A_1$ and $\sigma_2 = A_1$, respectively.

The examples show the difference in the growth rate of $D(\tau)$ due to the different variance of the particular noise. In the case of low variance, periodic undulation dominates the behaviour of $D(\tau)$. This leads to $H \cong 0$ for $\tau > 128$ on aggregate.

If the standard deviation of the noise is comparable with the amplitude of F_a, the influence of the noise totally shades the effect of periodicity in F_a.

In the actual series of the OLR, the annual amplitude is evidently higher than any other possible component of the OLR (Fig. 1a). Thus it would be interesting to estimate the growth rate for the deviations $F_r(t) = F(t) - F_a(t)$ (i.e deviations from the smooth annual cycle given by Eq. (3)). In principle, their growth rate may be considerably different from that of F_a. In the present section the difference in terms of H is crucial.

4. VARIATION OF $D(\tau)$ IN THE OLR (PENTAD) SERIES

Figure 2b shows the growth rate of $D(\tau)$ for two series of deviation (F_r). The following features can be observed.

1. Both curves of $D(\tau)$ have qualitatively similar shape. This is due to the functional relationship between F and T_e.
2. A tendency of decrease in $D(\tau)$, when the increment range approaches 14 years (i.e the term $Log_2(\tau)$ approaches 10) may be caused by the short data record.
3. The growth rate appears to be very low in comparison to the one, produced by the annual cycle for small values of τ. This means that the actual non-periodic influence to the global OLR flux density is weak, leading also to a low value of the corresponding exponent H. A similar tendency to saturation has been noticed earlier [10,12] in various air temperature time series. This result supports the assumption about domination of the negative feedback in the climate system for longer time scales than half a year. The latter criterion is determined by the length of the cycle of sign changes between the covariations in Eq. (6). An approximate value of the scale break can be found in Fig. 2a as the increment range, at which the rapid growth in $D(\tau)$ ends.

5. CONCLUSIONS

The aim of the paper was to estimate the sign of the cumulative feedback of the earth's climate system on the basis of the global average OLR record. The AVHRR-based series containing 5-day mean values was used for that purpose. The growth rate of the structure function was calculated over the increment range from 5 days to 14 years.

The OLR record has a remarkably strong annual cycle. Its annual amplitude exceeds the standard deviation of the residual variability. Thus the half length of the annual cycle determines the scale break caused by the covariations, having periodic changes of sign while τ increases. The break leads to a considerable decrease in the further growth rate of $D(\tau)$. The resulting very slow growth also produces a low value for $H \cong 0$, indicating the domination of negative feedback in the system, generating the time series [8].

The global average OLR flux density characterizes the response of the earth climate system to the outer influence. Thus its variability involves also the growing influence of the increase of the CO_2 concentration in the atmosphere. Figure 2a shows that such an increase would dominate only if two conditions were satisfied: (1) the concentration of CO_2 continues to increase and (2) its effect (to the OLR) remains statistically independent of the remaining variability of the climate system.

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Ühest Maa kliimasüsteemi tagasiside märgi hindamise võimalusest

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Mittestatsionaarse aegrea muudu teise momendi kasvukiirust muudu ulatuse funktsioonina saab kasutada seda aegrida genereerinud füüsikalise süsteemi tagasiside märgi hindamiseks. Artiklis on süsteemist aluspind + atmosfäär lahkuva soojuskiirguse voo 26 aasta pikkuse aegrea analüüsi abil näidatud, et kui reas on domineeriv periood, siis selle amplituud osutub tõkkeks, mis varjutab teised võimalikud mõjutused. Esitatud analüüs on rakendatav, kui on tarvis hinnata CO_2 kontsentratsiooni kasvu mõju kliimasüsteemi arengule.