
CHAPTER 7

INTEREST RATES AND BOND VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.
2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.
3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?
4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.
5. There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor's standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.
6. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly par.
7. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provide the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.
8. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.
9. Treasury bonds have no credit risk, so a rating is not necessary. Junk bonds often are not rated because there would no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).

10. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.
11. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.
12. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."
13. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.
14. One measure of liquidity is the bid-ask spread. Liquid instruments have relatively small spreads. Looking at Figure 7.4, the bellwether bond has a spread of one tick; it is one of the most liquid of all investments. Generally, liquidity declines after a bond is issued. Some older bonds, including some of the callable issues, have spreads as wide as six ticks.
15. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.
16. A hundred year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.

Solutions to Questions and Problems

Basic

1. The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.
2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.
3. $P = \$70(PVIFA_{9\%,10}) + \$1000(PVIF_{9\%,10}) = \$871.65$

B-70 SOLUTIONS

4. $P = \$1,075.25 = \$100(\text{PVIFA}_{R\%,9}) + \$1000(\text{PVIF}_{R\%,9})$; $R = \text{YTM} = 8.76\%$
5. $P = \$850 = \$C(\text{PVIFA}_{7.4\%,13}) + \$1000(\text{PVIF}_{7.4\%,13})$; $C = \$55.64$; coupon rate = 5.56%
6. $P = \$43.00(\text{PVIFA}_{3.75\%,20}) + \$1000(\text{PVIF}_{3.75\%,20}) = \$1,076.43$
7. $P = \$1,080 = \$39.00(\text{PVIFA}_{R\%,20}) + \$1000(\text{PVIF}_{R\%,20})$; $R = 3.345\%$; $\text{YTM} = 2 \times 3.345 = 6.69\%$
8. $P = \$850 = \$C(\text{PVIFA}_{4.5\%,29}) + \$1000(\text{PVIF}_{4.5\%,29})$; $C = \$35.64$; coupon rate = $2 \times 3.564 = 7.13\%$
9. Approximate = $.08 - .06 = .02$; Exact = $(1 + r)(1.06) - 1 = .08$; $r = 1.89\%$
10. $(1 + .035)(1 + .03) - 1 = 6.61\%$
11. $(1 + .10)(1 + h) = 1 + 0.16$; $h = 5.45\%$
12. $(1 + r)(1 + .04) = 1 + 0.13$; $r = 8.65\%$
13. This is a bond. Coupon rate = 7.50%. Bid price = $117:16 = 117.50\% \times \$1,000 = \$1,175.00$
 Previous day's asked price = today's asked price – change = $117 \frac{20}{32} + \frac{39}{32} = 118 \frac{27}{32}$
 $= 118.84375\% \times \$1,000 = \$1,188.4375$
14. This is a premium bond because it sells for more than 100% of face value.
 Current yield = $\$65/\$1,081.5625 = 6.01\%$; $\text{YTM} = 5.88\%$
 Bid-Ask spread = $118:05 - 118:03 = 2/32$

Intermediate

15. X: $P_0 = \$90(\text{PVIFA}_{7\%,13}) + \$1000(\text{PVIF}_{7\%,13}) = \$1,167.15$
 $P_1 = \$90(\text{PVIFA}_{7\%,12}) + \$1000(\text{PVIF}_{7\%,12}) = \$1,158.85$
 $P_3 = \$90(\text{PVIFA}_{7\%,10}) + \$1000(\text{PVIF}_{7\%,10}) = \$1,140.47$
 $P_8 = \$90(\text{PVIFA}_{7\%,5}) + \$1000(\text{PVIF}_{7\%,5}) = \$1,082.00$
 $P_{12} = \$90(\text{PVIFA}_{7\%,1}) + \$1000(\text{PVIF}_{7\%,1}) = \$1,018.69$; $P_{13} = \$1,000$
- Y: $P_0 = \$70(\text{PVIFA}_{9\%,13}) + \$1000(\text{PVIF}_{9\%,13}) = \850.26
 $P_1 = \$70(\text{PVIFA}_{9\%,12}) + \$1000(\text{PVIF}_{9\%,12}) = \856.79
 $P_3 = \$70(\text{PVIFA}_{9\%,10}) + \$1000(\text{PVIF}_{9\%,10}) = \871.65
 $P_8 = \$70(\text{PVIFA}_{9\%,5}) + \$1000(\text{PVIF}_{9\%,5}) = \922.21
 $P_{12} = \$70(\text{PVIFA}_{9\%,1}) + \$1000(\text{PVIF}_{9\%,1}) = \981.65 ; $P_{13} = \$1,000$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. In both cases, the largest percentage price changes occur at the shortest maturity lengths.

16. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 8 percent. If the YTM suddenly rises to 10 percent:

$$P_{\text{Bob}} = \$40(\text{PVIFA}_{5\%,4}) + \$1000(\text{PVIF}_{5\%,4}) = \$964.54$$

$$P_{\text{Tom}} = \$40(\text{PVIFA}_{5\%,30}) + \$1000(\text{PVIF}_{5\%,30}) = \$846.28$$

$$\Delta P_{\text{Bob}}\% = (\$964.54 - 1000) / \$1000 = -3.55\%$$

$$\Delta P_{\text{Tom}}\% = (\$846.28 - 1000) / \$1000 = -15.37\%$$

If the YTM suddenly falls to 4 percent:

$$P_{\text{Bob}} = \$40(\text{PVIFA}_{3\%,4}) + \$1000(\text{PVIF}_{3\%,4}) = \$1,037.17$$

$$P_{\text{Tom}} = \$40(\text{PVIFA}_{3\%,30}) + \$1000(\text{PVIF}_{3\%,30}) = \$1,196.00$$

$$\Delta P_{\text{Bob}}\% = (\$1,037.17 - 1000) / \$1000 = +3.72\%$$

$$\Delta P_{\text{Tom}}\% = (\$1,196.00 - 1000) / \$1000 = +19.60\%$$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

17. Initially, at a YTM of 8 percent, the prices of the two bonds are:

$$P_J = \$25(\text{PVIFA}_{4\%,16}) + \$1000(\text{PVIF}_{4\%,16}) = \$825.22$$

$$P_K = \$55(\text{PVIFA}_{4\%,16}) + \$1000(\text{PVIF}_{4\%,16}) = \$1,174.78$$

If the YTM rises from 8 percent to 10 percent:

$$P_J = \$25(\text{PVIFA}_{5\%,16}) + \$1000(\text{PVIF}_{5\%,16}) = \$729.06$$

$$P_K = \$55(\text{PVIFA}_{5\%,16}) + \$1000(\text{PVIF}_{5\%,16}) = \$1,054.19$$

$$\Delta P_J\% = (\$729.06 - 825.22) / \$825.22 = -11.65\%$$

$$\Delta P_K\% = (\$1,054.19 - 1,174.78) / \$1,174.78 = -10.26\%$$

If the YTM declines from 8 percent to 6 percent:

$$P_J = \$25(\text{PVIFA}_{3\%,16}) + \$1000(\text{PVIF}_{3\%,16}) = \$937.19$$

$$P_K = \$55(\text{PVIFA}_{3\%,16}) + \$1000(\text{PVIF}_{3\%,16}) = \$1,314.03$$

$$\Delta P_J\% = (\$937.19 - 825.22) / \$825.22 = +13.57\%$$

$$\Delta P_K\% = (\$1,314.03 - 1,174.78) / \$1,174.78 = +11.85\%$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18. $P_0 = \$1,040 = \$50(\text{PVIFA}_{R\%,14}) + \$1000(\text{PVIF}_{R\%,14})$; $R = 4.606\%$, $\text{YTM} = 2 \times 4.606\% = 9.21\%$
Current yield = $\$100/\$1,040 = 9.62\%$; effective annual yield = $(1.04606)^2 - 1 = 9.42\%$

19. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

$$P = \$1,095 = \$40(\text{PVIFA}_{R\%,20}) + \$1000(\text{PVIF}_{R\%,20}); R = 3.34\%; \text{YTM} = 2 \times 3.34\% = 6.68\%$$

20. Current yield = $.0980 = \$120/P_0$; $P_0 = \$120/.0980 = \$1,224.49$

$$P_0 = \$1,224.49 = \$120[(1 - (1/1.09)^N) / .09] + \$1,000/1.09^N$$

$$1,224.49(1.09)^N = 1,333.33(1.09)^N - 1,333.33 + 1,000$$

$$333.33 = 108.84(1.09)^N; 3.0625 = 1.09^N; N = \log 3.0625 / \log 1.09 = 13 \text{ years}$$

21. Current yield = $.094 = \$78.75/P_0$; $P_0 = \$78.75/.094 = \$837.77 = 83.77\%$ of par $\approx 83\frac{3}{4}$

Bond closed down $\frac{1}{2}$, so yesterday's close = $83\frac{3}{4} + \frac{1}{2} = 84\frac{1}{4}$

B-72 SOLUTIONS

22. *a.* Bond price is the present value term when valuing the cash flows from a bond; YTM is the interest rate used in valuing the cash flows from a bond.
- b.* If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.
- c.* Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.
23. *a.* $P_0 = \$1,000/1.09^{20} = \178.43
- b.* $P_1 = \$1,000/1.09^{19} = \194.49 ; year 1 interest deduction = $\$194.49 - 178.43 = \16.06
 $P_{19} = \$1,000/1.09 = \917.43 ; year 19 interest deduction = $\$1,000 - 917.43 = \82.57
- c.* Total interest = $\$1,000 - \$178.43 = \$821.57$
 Annual interest deduction = $\$821.57/20 = \41.08
- d.* The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.
24. *a.* The coupon bonds have a 9% coupon which matches the 9% required return, so they will sell at par; # of bonds = $\$10M/\$1,000 = 10,000$.
 For the zeroes, $P_0 = \$1,000/1.09^{30} = \75.37 ; $\$10M/\$75.37 = 132,679$ bonds will be issued.
- b.* Coupon bonds: repayment = $10,000(\$1,090) = \$10.9M$
 Zeroes: repayment = $132,679(\$1,000) = \$132,679,000$
- c.* Coupon bonds: $(10,000)(\$90)(1-.35) = \$585,000$ cash outflow
 Zeroes: $P_1 = \$1,000/1.09^{29} = \82.15 ; year 1 interest deduction = $\$82.15 - 75.37 = \6.78
 $(132,679)(\$6.78)(.35) = \$314,847.27$ cash inflow
 During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.
25. The maturity is indeterminate; a bond selling at par can have any length of maturity.

B-74 SOLUTIONS

Solve for

8.76%

5.

Enter	13	7.4%	-\$850		\$1,000
	N	I/Y	PV	PMT	FV
Solve for				\$55.64	

$$\text{Coupon rate} = \$55.64 / \$1,000 = 5.56\%$$

6.

Enter	20	3.75%		\$43	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,076.43		

7.

Enter	20		-\$1,080	\$39	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		3.345%			

$$3.345\% \times 2 = 6.69\%$$

8.

Enter	29	4.5%	-\$850		\$1,000
	N	I/Y	PV	PMT	FV
Solve for				\$35.64	

$$\$35.64 \times 2 = \$71.28; \$71.28 / \$1,000 = 7.13\%$$

15. Bond X

P_0

Enter	13	7%		\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,167.15		

P_1

Enter	12	7%		\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,158.85		

P_3

Enter	10	7%		\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,140.47		

P_8

Enter	5	7%		\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,082.00		

B-76 SOLUTIONS

P_{12}					
Enter	1	7%		\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,018.69		

Bond Y

P_0					
Enter	13	9%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$850.26		

P_1					
Enter	12	9%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$856.79		

P_3					
Enter	10	9%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$871.65		

P_8					
Enter	5	9%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$922.21		

P_{12}					
Enter	1	9%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$981.65		

16. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 8 percent. If the YTM suddenly rises to 10 percent:

P_{Bob}					
Enter	4	5%		\$40	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$964.54		
	$\Delta P_{\text{Bob}}\% = (\$964.54 - 1000) / \$1000 = -3.55\%$				

P_{Tom}					
Enter	30	5%		\$40	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$846.28		
	$\Delta P_{\text{Tom}}\% = (\$846.28 - 1000) / \$1000 = -15.37\%$				

If the YTM suddenly falls to 6 percent:

P_{Bob}					
Enter	4	3%		\$40	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,037.17		
	$\Delta P_{\text{Bob}}\% = (\$1,037.17 - 1000) / \$1000 = + 3.72\%$				

P_{Tom}					
Enter	30	3%		\$40	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,196.00		
	$\Delta P_{\text{Tom}}\% = (\$1,196.00 - 1000) / \$1000 = + 19.60\%$				
	All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.				

17. Initially, at a YTM of 8 percent, the prices of the two bonds are:

P_{J}					
Enter	16	4%		\$25	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$852.22		

P_{K}					
Enter	16	4%		\$55	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,174.78		

If the YTM rises from 8 percent to 10 percent:

P_{J}					
Enter	16	5%		\$25	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$729.06		
	$\Delta P_{\text{J}}\% = (\$729.06 - 825.22) / \$825.22 = - 11.65\%$				

P_{K}					
Enter	16	5%		\$55	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,054.19		
	$\Delta P_{\text{K}}\% = (\$1,054.19 - 1,174.78) / \$1,174.78 = - 10.26\%$				

If the YTM declines from 8 percent to 6 percent:

P_{J}					
Enter	16	3%		\$25	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$937.19		
	$\Delta P_{\text{J}}\% = (\$937.19 - 825.22) / \$825.22 = + 13.57\%$				

B-78 SOLUTIONS

P_K

Enter	16	3%		\$55	\$1,000
	N	I/Y	PV	PMT	FV

Solve for $\$1,314.03$

$$\Delta P_K\% = (\$1,314.03 - 1,174.78) / \$1,174.78 = + 11.85\%$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18.

Enter	14		-\$1,040	\$50	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 4.606%

$$4.606\% \times 2 = 9.21\%$$

19. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter	20		-\$1,095	\$40	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 3.34%

$$3.34\% \times 2 = 6.68\%$$

20. Current yield = .0980 = $\$120/P_0$; $P_0 = \$120/.0980 = \$1,224.49$

Enter		9%	-\$1,224.49	\$120	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 12.96
12.96 or ≈ 13 years

23.

a. P_0

Enter	20	9%			\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$178.43

b. P_1

Enter	19	9%			\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$194.49
year 1 interest deduction = $\$194.49 - 178.43 = \16.06

P_{19}

Enter	1	9%			\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$917.43
year 19 interest deduction = $\$1,000 - 917.43 = \82.57

B-80 SOLUTIONS

P₁

Enter	7	8%		\$60	\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$895.87

Current yield = $\$60 / \$885.07 = 6.78\%$

Capital gains yield = $(\$895.87 - 885.07) / \$885.07 = + 1.22\%$

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 8%, but this return is distributed differently between current income and capital gains.

27.

a.

Enter	10		-\$1,150	\$90	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 6.88%

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

Enter	8	5.88%		\$90	\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$1,194.91

The HPY is:

Enter	2		-\$1,150	\$90	\$1,194.91
	N	I/Y	PV	PMT	FV

Solve for 9.69%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates have dropped by 1 percent; bond prices rise when yields fall.

28.

P_M

CF₀	\$0
C01	\$0
F01	12
C02	\$1,000
F02	16
C03	\$1,750
F03	11
C04	\$21,750
F04	1

I = 6%

NPV CPT

\$9,837.00

P_N

Enter

40

6%

\$20,000

N

I/Y

PV

PMT

FV

Solve for

\$1,944.44