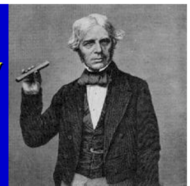
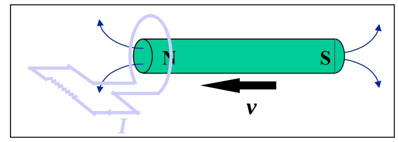
Faraday's Law



Chapter 31

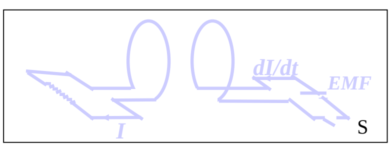
Law of Induction (emf)
Faraday's Law
Magnetic Flux
Lenz's Law
Generators
Induced *Electric* fields

Faraday's Experiments



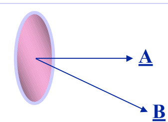
- Michael Faraday discovered induction in 1831.
- Moving the magnet induces a current I.
- Reversing the direction reverses the current.
- Reversing the magnet reverses the currents.
- The induced current is set up by an *induced EMF*.

Faraday's Experiments



- Changing the current in the right-hand coil induces a current in the left-hand coil.
- The induced current does not depend on the size of the current in the right-hand coil;
- It depends on *dI/dt* .

Magnetic Flux

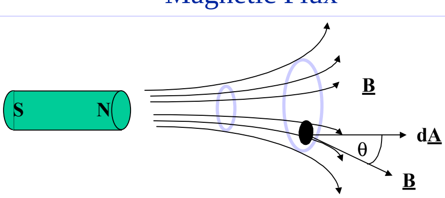


- These very different appearing cases can be united by the concept of *magnetic flux*.
- In the easiest case, with a constant **B** and a flat surface of area area, the magnetic flux is

$$\Phi_B = \vec{B} \bullet \vec{A}$$

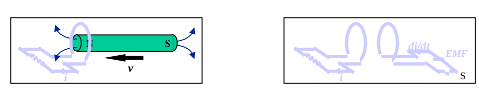
Units : 1 tesla x m^2 = 1 weber

Magnetic Flux



- When B is not constant, or the surface is not flat, one must do an integral.
- Break the surface into bits d**A**. The flux through one bit is $d\Phi_B = \mathbf{B} \bullet d\mathbf{A} = B\, dA \cos\theta$.
- Add them: $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos\theta \, dA$

Faraday's Law



- Moving the magnet changes the flux Φ_B .
- Change current changes the flux Φ_B .
- **And c**hanging the flux induces an emf:

$$emf = -\, d\Phi_B / dt$$

Faraday's law

The emf induced around a loop **equals the rate of change of the flux through that loop**

Lenz's Law

- Faraday's law gives the direction of the induced emf and therefore of any induced current.
- Lenz's law is a simple way to get the directions straight with less effort.
- Lenz's Law:

The induced emf is directed so that any resulting induced current flow will *oppose* the *change* in magnetic flux which causes the induced emf.

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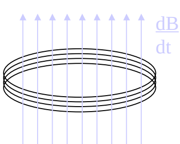
The induced emf is directed so that any resulting induced current flow will *oppose* the *change* in magnetic flux which causes the induced emf.

Lenz's Law: Nobody wants to be FLUXED with.

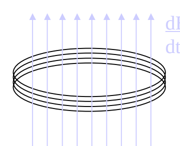
Example of Faraday's Law

Consider a coil of radius 5 cm with N = 250 turns.
A magnetic field through it changes at the rate of
$dB/dt = 0.6$ T/s.
The total resistance of the coil is 8 Ω.
What is the induced current ?



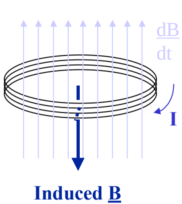
Use Lenz's law to determine the direction of the induced current.
Apply Faraday's law to find the emf and then the current.

Example of Faraday's Law



Lenz's law:

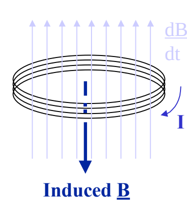
Example of Faraday's Law



Lenz's law:
The change in B is increasing the upward flux through the coil.
So the induced current will have a magnetic field whose flux (and therefore field) is *down*.

Hence the induced current must be *clockwise* when looked at from above.

Now use Faraday's law to get the magnitude of the induced emf and current.



The induced EMF is

$$emf = - d\Phi_B /dt$$

In terms of B: $\Phi_B = N(BA)$
$= NB(\pi r^2)$

Therefore $emf = - N (\pi r^2)\, dB/dt$

$emf = - (250)(\pi\, 0.005^2)(0.6T/s) = -1.18$ V

($1V=1Tm^2/s$)

Current $I = |emf|/R = (1.18V) / (8\ \Omega) = 0.147$ A

Type of Problems with Faraday's Law

$\Phi_B = \mathbf{B} \bullet \mathbf{A} = B\,A\cos(\theta)$ and emf $= -d\Phi_B/dt$

For the emf to be non-zero one (or more) of three things must be changing in time: B, A, or θ! Thus, there are three types of problems.

1. In the previous example B was changing in time, dB/dt.

2. θ changing in time. If $d\theta/dt = \omega$, then the result is an AC generator.

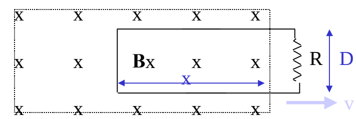
3. And, finally the Area, A, can change in time.

Motional EMF

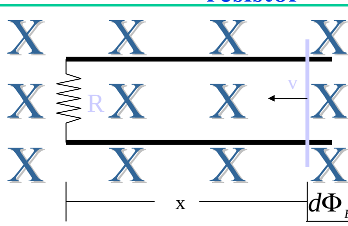
Up until now we have considered fixed loops. The flux through them changed because the magnetic field changed with time.

Now try moving the loop in a uniform and constant magnetic field. This changes the flux, too.

B points into screen



Example: Find the current in the resistor



$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

$$\Phi_B = \vec{B} \bullet \vec{A} = BA$$

$$\Phi_B = Bxy$$

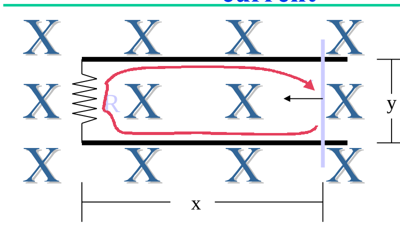
$$\frac{d\Phi_B}{dt} = \frac{d}{dt}Bxy = By\frac{dx}{dt}$$

Where, R = 5 Ω
y = 1.5 m
x = 3.6 m
v = 2.5 m/s
B = 3 T

$$\frac{d\Phi_B}{dt} = Byv \qquad emf = -\frac{d\Phi_B}{dt} = -Byv$$

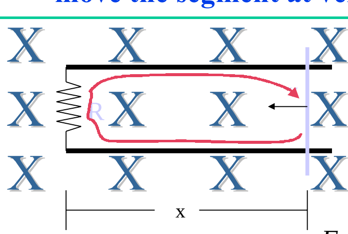
$$i = \frac{|emf|}{R} = \frac{|Byv|}{R} = 2.25\text{ A}$$

Example: Find the direction of the current



Where, R = 5 Ω
y = 1.5 m
x = 3.6 m
v = 2.5 m/s
B = 3 T

Example: Find the force required to move the segment at velocity, v.



$$\text{Power} = \vec{F} \bullet \vec{v}$$

$$\text{Power} = i^2R$$

$$F = \frac{\text{Power}}{v}$$

$$F = \frac{i^2R}{v} = 10.125\text{ N}$$

Where, R = 5 Ω
y = 1.5 m
x = 3.6 m
v = 2.5 m/s
B = 3 T

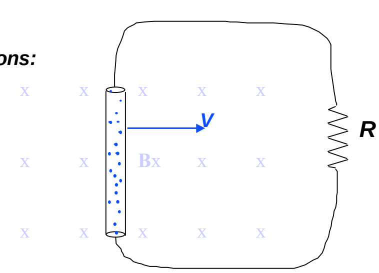
But the force is on the moving wire, not the resistor!

$F = i\vec{L} \times \vec{B} = iyB$???

Motional EMF

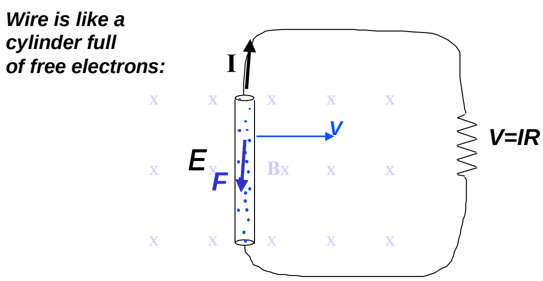
Remember what is happening when a conductor is moved through a field:

Wire is like a cylinder full of free electrons:



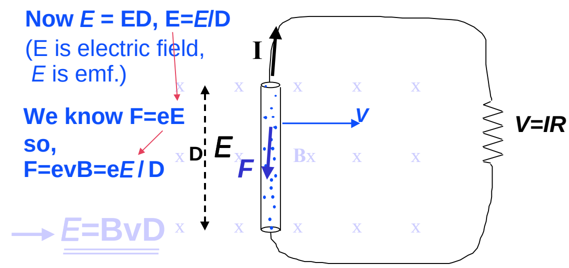
Motional EMF

Remember what is happening when a conductor is moved through a field:

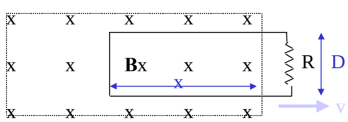


Motional EMF

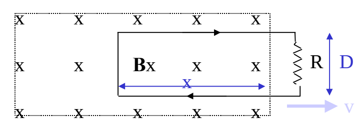
Remember what is happening when a conductor is moved through a field:



Now use Faraday's Law:



Faraday's Law:



The flux is $\Phi_B = \underline{B}\cdot\underline{A} = BDx$

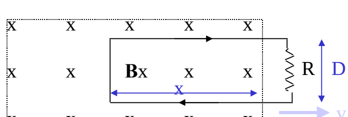
This changes in time:

$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$

Hence by Faraday's law there is an induced emf and current. What direction is it?

Lenz's law: there is less inward flux through the loop. Hence the induced current gives inward flux.

So the induced current is clockwise.



Now Faraday's Law $d\Phi_B/dt = -E$ gives the EMF:

$E = BDv$

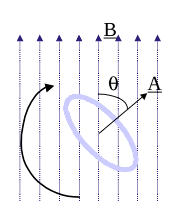
In a circuit with a resistor, this gives

$E = BDv = IR$: $I = BDv/R$

Thus moving a circuit in a magnetic field produces an emf exactly like a battery. This is the principle of an electric generator.

Rotating Loop - The Generator

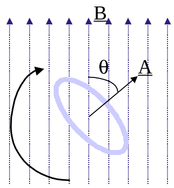
Consider a loop of area A in a region of space in which there is a uniform magnetic field B.
Rotate the loop with an angular frequency ω .



Rotating Loop - The Generator

Consider a loop of area A in a region of space in which there is a uniform magnetic field B.
Rotate the loop with an angular frequency ω .

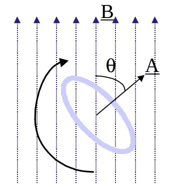
The flux changes because angle θ changes with time: θ = ωt.



Hence:

1. $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos(\theta)$
   $= BA\cos(\omega t)$

2. $d\Phi_B/dt = d(BA\cos(\omega t))/dt$
   $= BA\, d(\cos(\omega t))/dt$
   $= - BA\omega \sin(\omega t)$



$d\Phi_B/dt = - BA\omega \sin(\omega t)$

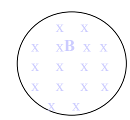
3.

- Then by Faraday's Law this motion causes an emf
  $E = - d\Phi_B /dt = BA\omega \sin(\omega t)$
- This is an AC (alternating current) generator.

A new source of EMF

- If we have a conducting loop in a magnetic field, we can create an EMF (like a battery) by changing the value of B•A .
- This can be done by changing the area, by changing the magnetic field, or both.
- We can use this source of EMF in electrical circuits in the same way we used batteries.
- Remember we have to do work (kinetic energy) to move the loop or change B, to generate EMF. (Nothing is for free!!)

Example: A circular UHF TV antenna has a diameter of 11.2 cm. The magnetic field of a TV signal is normal to the plane of the loop, and at any instant in time its magnitude is changing at the rate of 157 mT/s. What is the EMF ?



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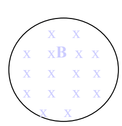
Magnetic flux:



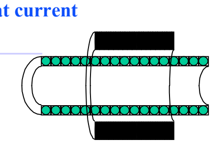
$$\Phi_B = \int \bar{B} \bullet d\bar{A} = \int BdA\cos\theta$$

Induced EMF:

$$E = -N\left[\frac{d\Phi_B}{dt}\right] = N\left[\frac{d(0.25B\pi D^2)}{dt}\right] = N(0.25\pi D^2)\frac{dB}{dt}$$

$$= (1)[0.24\pi(0.112m)^2](0.157T/s)$$

Example: a 120 turn coil (r= 1.8 cm, R = 5.3Ω) is placed outside a solenoid,(r=1.6cm, n=220/cm, i=1.5A). The current in the solenoid is reduced to 0 in 0.16s. What current appears in the coil ?



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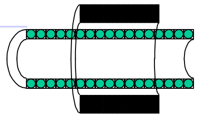


Current induced in coil i_c

$$i_C = \frac{EMF}{R} = \left(\frac{N}{R}\right)\frac{d\Phi_B}{dt}$$

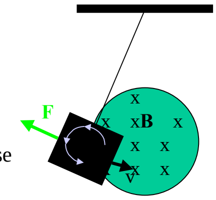
$$\Phi_B = \int \underline{B} \bullet \underline{dA} = \mu_0 n i_s A_s$$

$$i_c = \left(\frac{N}{R}\right)\frac{d(\mu_0 n i_s A_s)}{dt} = \left(\frac{N}{R}\right)\mu_0 n A_s \frac{di_s}{dt} = \left(\frac{N}{R}\right)\mu_0 n A \frac{i_0}{t}$$

$$i_c = 5.97mA$$

Eddy Currents

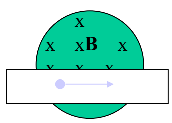
If you move any conductor through a magnetic field, then you induce a current in the conductor.



- Circulating currents are called "eddy" currents.
- Because the resistance is small, these currents can be large.
- Currents dissipate energy in conductor.
- Motion of current is such as to produce force opposing motion.
- Can be used as a "fail-safe" braking system.
- Can be undesirable. Laminations in cores of electric motors minimize eddy currents and reduce heat buildup.

Induced electric fields

Consider a conductor in a time-varying magnetic field. When we induce a current in the conductor,



- **the free charges in the conductor must then experience a force.**
- **The force on a charge is q<u>E.</u>**
- **This field is called <u>induced electric field</u>**
  - *What <u>kind</u> of field is this?*

Induced electric fields

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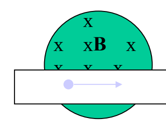
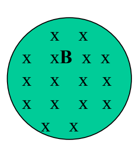


- **Work done in inducing field must be $\oint$ <u>E</u>.<u>dl</u>**
- **Thus** $E = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A} = \oint \vec{E}\cdot d\vec{\ell}$
- **The <u>induced</u> electric field is not conservative!**

Induced electric fields

Consider a conductor in a time-varying magnetic field
- with no conductor present:



- is there now an electric field?

Induced electric fields

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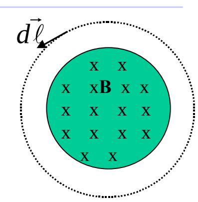


- is there now an electric field?

- YES! - the field is only *felt* by the charges in a conductor, not *caused* by them.

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A} = \oint \vec{E}\cdot d\vec{\ell}$$

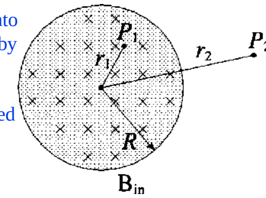
$emf = -\, d\Phi_B / dt$

Faraday's law

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

with $\oint \vec{E} \cdot d\vec{\ell} = emf$

Example: A magnetic field directed into a circular region of the board is given by B = Csin(ωt), where C = 3 T, ω = 30 rad/s and R = 0.4 m.
a. What is the magnitude of the induced electric field at P_1, r_1=0.2 m at t=1.2s?
b. What is the distance, r_2, if $E_1 = E_2$?



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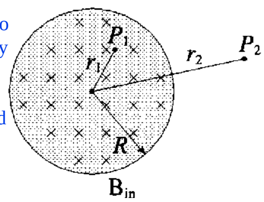

a. Apply Faraday's law

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

By symmetry, E is constant and circular at a fixed radius.

$$\oint \vec{E} \cdot d\vec{\ell} = E_1\, 2\pi r_1$$

Example: A magnetic field directed into a circular region of the board is given by B = Csin(ωt), where C = 3 T, ω = 30 rad/s and R = 0.4 m.
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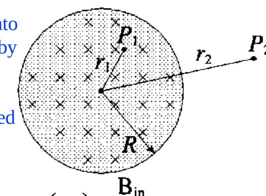

$$\oint \vec{E} \cdot d\vec{\ell} = E_1\, 2\pi r_1 = -\pi r_1^2 C\omega\cos(\omega t)$$

Solve the RHS → $-d\Phi_B/dt$
Φ first, then $d\Phi_B/dt$, then finish

$$E_1 = -\tfrac{1}{2} r_1 C\omega\cos(\omega t)$$

$$\Phi_B = \vec{B} \bullet \vec{A} = B\pi r_1^2$$

$$\frac{d\Phi_B}{dt} = \pi r_1^2 \frac{dB}{dt} = \pi r_1^2 \frac{d\left(C\sin(\omega t)\right)}{dt} = \pi r_1^2 C\omega\cos(\omega t)$$

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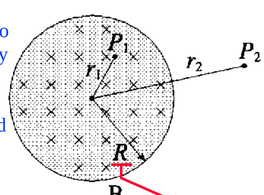


$$E_1 = -\tfrac{1}{2} r_1 C\omega\cos(\omega t)$$

$$E_1 = -\tfrac{1}{2} 0.2m(3\text{T})30\tfrac{rads}{s}\cos(30\tfrac{rads}{s}1.2s) = 1.15\,\tfrac{\text{N}}{\text{C}}$$

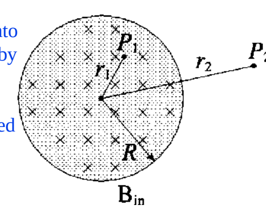
b. Apply Faraday's law at P_2

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = B\pi R^2$$

$$E_2 2\pi r_2 = -\pi R^2 \frac{dB}{dt} = -\pi R^2 C\omega\cos(\omega t)$$

$$E_2 = -\tfrac{1}{2}\frac{R^2 C\omega}{r_2}\cos(\omega t)$$

Example: A magnetic field directed into a circular region of the board is given by B = Csin(ωt), where C = 3 T, ω = 30 rad/s and R = 0.4 m.
a. What is the magnitude of the induced electric field at P_1, r_1=0.2 m at t=1.2s?
b. What is the distance, r_2, if $E_1 = E_2$?



$$E_1 = -\tfrac{1}{2} r_1 C\omega\cos(\omega t)$$

$$E_2 = -\tfrac{1}{2}\frac{R^2 C\omega}{r_2}\cos(\omega t) \quad \rightarrow r_1 = \frac{R^2}{r_2}$$

$$\rightarrow r_2 = \frac{R^2}{r_1} = \frac{0.4^2}{0.2}m = 0.8\,m$$