Einstein's Relativity

Chapter 39

Time dilation
Length contraction
Dynamics

Einstein's Postulates of Relativity

1. All laws of physics are the same in all inertial frames.

2. The speed of light in a vacuum has the same value, c, in all inertial reference frames.

Results of the Postulates

1. All laws of physics are the same in all inertial frames.

What we have studied about physics is valid, at least this semester!

If we repeat our lab experiments on another planet or in an inertial rocket ship moving through space, the results will be the same or explainable in terms of what we know.

Results of the Postulates

2. The speed of light in a vacuum has the same value, c, in all inertial reference frames.

Loss of simultaneity

Time dilation
Length contraction
Invariant form of the light cone in the space time diagram.

Results of the Postulates and Relativity

Define gamma: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Time dilation $\Delta t = \gamma \Delta t'$

Objects moving relative to us have their clocks (internal processes) slowed by a factor of gamma.

Example: muons produced via cosmic-rays.

Results of the Postulates and Relativity

Define gamma: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Length contraction $\Delta L = \gamma^{-1} \Delta L'$

Objects moving relative to us have their lengths in the directions of motion contracted by a factor of gamma.

Example: muons produced via cosmic-rays.
Example: as one approaches the speed of light, they compress lengthwise to nothing.

Special Relativity: Example

A spaceship observes clocks on earth running at a rate of 0.50 of that of the spaceships clock.
a. How fast is the earth moving with respect to the spaceship?
b. How fast is the spaceship moving with respect to the earth?

Special Relativity: Example

A spaceship observes clocks on earth running at a rate of 0.50 of that of the spaceships clock.
a. How fast is the earth moving with respect to the spaceship?
b. How fast is the spaceship moving with respect to the earth?

$$t = \gamma\, t' = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \frac{t'}{t} = \sqrt{1-\frac{v^2}{c^2}}, \quad \left(\frac{t'}{t}\right)^2 = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\left(\frac{t'}{t}\right)^2, \quad v^2 = c^2\left(1-\left(\frac{t'}{t}\right)^2\right) \quad v = c\left(1-\left(\frac{t'}{t}\right)^2\right)^{1/2} = c\left(1-.5^2\right)^{1/2} = 0.866c$$

a. v = 0.866c, b. v = 0.866c

Special Relativity: Example

A 100 m spaceship passes an observer on earth. The observer on earth finds the spaceship to take 200 ns to pass overhead.
a. How fast is the spaceship moving?
b. What length does the observer measure the spaceship?

$$v = \frac{\text{Length}}{\text{time}}$$

$$\text{Length} = \frac{\text{Proper Length}}{\gamma}$$

$$v = \frac{\text{Proper Length}}{\gamma \ \ \text{time}}$$

$$v = \frac{\text{Proper Length}}{\gamma \ \ \text{time}} = \frac{\text{L}\ \sqrt{1-\frac{v^2}{c^2}}}{t}$$

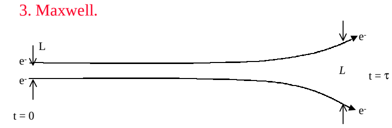
$$v^2 = \frac{\text{L}^2\left[1-\frac{v^2}{c^2}\right]}{t^2}, \quad v^2\left(\frac{t^2}{\text{L}^2}+\frac{1}{c^2}\right) = 1$$

$$v = \left(\frac{t^2}{\text{L}^2}+\frac{1}{c^2}\right)^{-\frac{1}{2}} = \left(\frac{(2x10^{-7})^2}{100^2}+\frac{1}{(3x10^8)^2}\right)^{-\frac{1}{2}}$$

a. v = 0.87c b. Gamma = ?, L = L_o/gamma

Relativity

Example: Two electrons move along the x-axis at 0.99c from the point of view of the rest frame. If they are separated by L = 2.000 µm in the y direction at t = 0, how long will it take for the two electrons to be separated by *L* = 2.001 µm in the y direction from the point of view of the observer in the rest frame. Answer from the point of view of
1. Newton,
2. Einstein, and
3. Maxwell.



Special Relativity: Dynamics

$$p = \gamma m v$$

$$E_{total} = KE + m_o c^2 = \gamma \ m_o c^2$$

Special Relativity: Example

Find the momentum (in MeV/c) and kinetic energy (in MeV) of an electron traveling at 0.900c.

p = γmv, and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-.9^2}} = 2.29$

$m_e = 9.1x10^{-31}kg = 9.1x10^{-31}kg \times (3x10^8 m/s)^2/c^2 \times (1.6 \times 10^{-13}\ J/MeV)^{-1} = 0.511\ MeV/c^2$

$p = \gamma mv = 2.29 \times 0.511\ MeV/c^2 \times 0.9c = 1.05\ MeV/c$

$E = \gamma mc^2 = 2.29 \times 0.511\ MeV/c^2 \times c^2 = 1.17\ MeV = KE + mc^2$
$KE = E - mc^2 = (1.17 - 0.51)\ MeV = 0.66\ MeV$