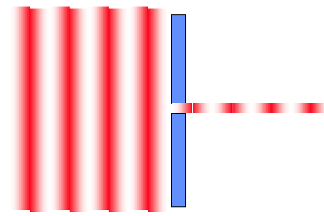
Diffraction and Polarization

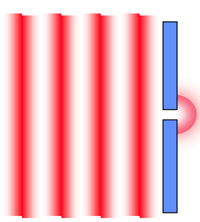
Chapter 38
Diffraction
Rayleigh's Criterion
Polarization

Diffraction

In geometrical optics, we modeled rays like this!



Diffraction

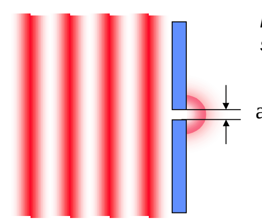


In fact what happens is this... A spherical wave propagates out from the aperture.

All waves do this ..

For double slit interference we did this without considering the size of the aperture.

Diffraction

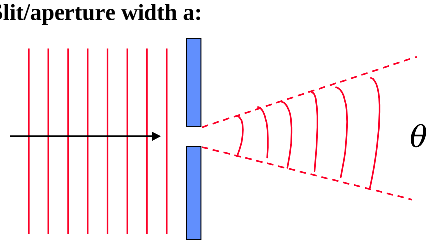


Diffraction considers the size of the aperture, a.

Diffraction - Effects due to the fact that apertures are not perfect point sources, but slits with finite width. Waves emerging from different locations in the aperture will interfere with each other.

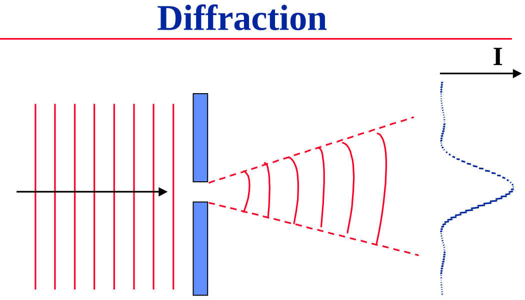
Diffraction

Slit/aperture width a:



Angular Spread: $\theta \sim \lambda/a$

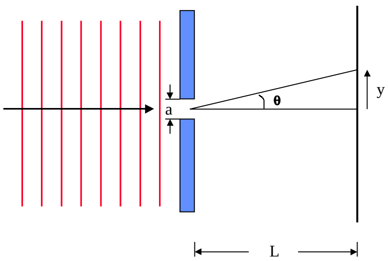
Diffraction



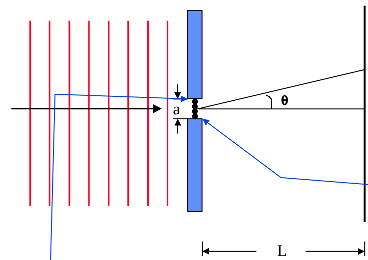
Angular Spread proportional to λ/a

(Actual intensity distribution)

Diffraction

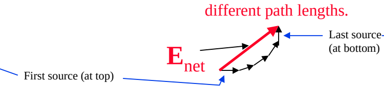


Diffraction

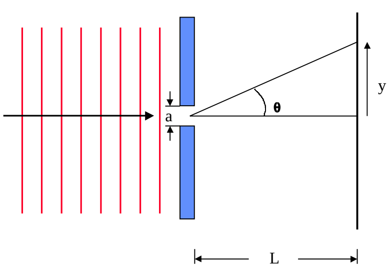


Consider the aperture to be a large collection of point sources which add together at the screen. The more sources there are the weaker each source must be.

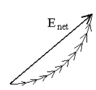
At y the electric field is the sum of each of many (weak) sources all with different path lengths.



Diffraction

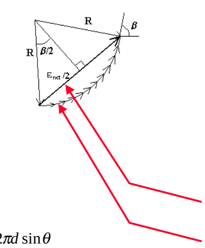


In a real aperture, the number of point sources approaches infinity, while each one's strength simultaneously goes to zero.



This sum of infinitesimal sources forms an arc. The sum of which is the straight line connecting the end points, E_{net}.

Diffraction



This sum of infinitesimal sources forms an arc. The sum of which is the straight line connecting the end points, E_{net}.

β = angle between the first and last infinitesimal electric field. **β** is exactly **ϕ** for the double slit experiment with d equal to the aperture size a!!!

The arc of electric field has some radius R.

$E_{net}/2 = R \sin(\beta/2)$, R = ?

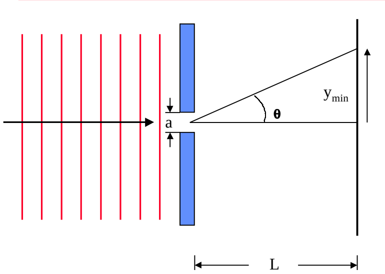
$E_{max} = R\,\beta$

$$\phi = \frac{2\pi d \sin\theta}{\lambda}$$

$$\beta = \frac{2\pi a \sin\theta}{\lambda}$$

Thus, $$E_{net} = E_{\max}\frac{\sin(\beta/2)}{(\beta/2)}$$

Diffraction



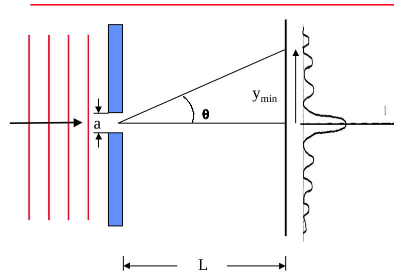
$$\beta = \frac{2\pi a \sin\theta}{\lambda}$$

$$E_{net} = E_{\max}\frac{\sin(\beta/2)}{(\beta/2)}$$

$$I \propto E^2, \quad so$$

$$I_{net} = I_{\max}\frac{\sin^2(\beta/2)}{(\beta/2)^2}$$

Diffraction



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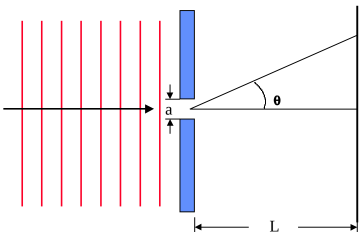
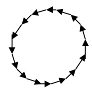


y will be a minimum (dim) when the net sum E field is zero. Actually, we have an infinite amount of sources. We get zero intensity on the screen when $E_{net} = 0$.

$E_{net} = 0$

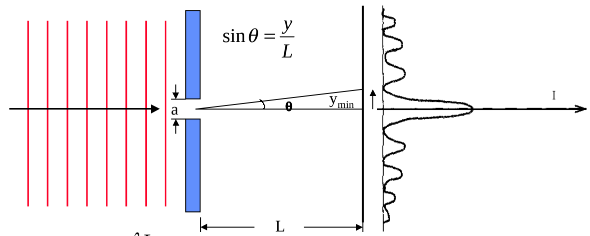


This means that the first source (the one at the top) and the last source (the one at the bottom) have parallel E field (constructive)!
This is exactly the case for a maximum for two source interference separated now by a, not d.

Y_{max} (double source) $\rightarrow Y_{min}$(single) and d $\rightarrow$ a

Diffraction

$\sin\theta = \frac{y}{L}$

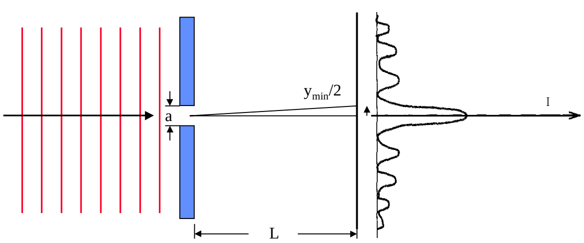


$$y_{min} = m\frac{\lambda L}{a}, \quad m \neq 0$$

$$y_{max} = 0$$

Minimums where $\sin\theta = m\frac{\lambda}{a}, \quad m \neq 0$

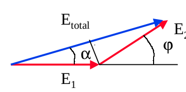
Diffraction



Most of the intensity of the light is between $+y_{min}/2$ and $- y_{min}/2$. Thus

$$\sin\theta \cong \theta \cong \frac{\lambda}{a} \quad \textit{the angular spread}$$

Young's Double Slit Experiment



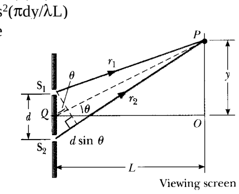
$$\phi = kd\sin\theta = \frac{2\pi}{\lambda}d\sin\theta$$

$$\frac{\phi}{2\pi} = \frac{d\sin\theta}{\lambda} \cong \frac{dy}{\lambda L}$$

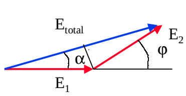
$\alpha = \phi/2$, $|E_1| = |E_2| = E$

$E_{total} = 2E\cos(\alpha) = 2E\cos(\phi/2) = 2E\cos(\pi\, d\sin(\theta)/\lambda)$
$I \propto E_{total}^2 \rightarrow I = 4I_1\cos^2(\pi\, d\sin(\theta)/\lambda) \cong 4I_1\cos^2(\pi dy/\lambda L)$
where I_1 is the intensity at S_1 (or S_2, they are same).

$I = I_o\cos^2(\pi\, d\sin(\theta)/\lambda) \cong I_o\cos^2(\pi\, dy/\lambda L)$
Where I_o is the peak intensity at the screen



Young's Double Slit Experiment



$$\phi = kd\sin\theta = \frac{2\pi}{\lambda}d\sin\theta$$

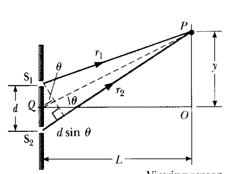
$$\frac{\phi}{2\pi} = \frac{d\sin\theta}{\lambda} \cong \frac{dy}{\lambda L}$$

$\alpha = \phi/2$, $|E_1| = |E_2| = E$
$I = I_o\cos^2(\pi\, d\sin(\theta)/\lambda) \cong I_o\cos^2(\pi\, dy/\lambda L)$
Where I_o is the peak intensity at the screen

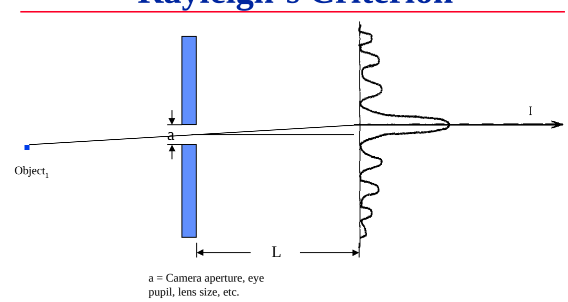
Add diffraction -

$$I_{net} = I_{max}\frac{\sin^2(\beta/2)}{(\beta/2)^2}\cos^2(\phi/2)$$

$$\phi = \frac{2\pi d\sin\theta}{\lambda} \qquad \beta = \frac{2\pi a\sin\theta}{\lambda}$$

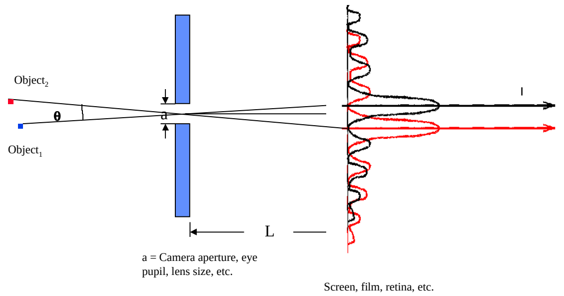


Rayleigh's Criterion



First object produce a diffracted object on the screen.

Rayleigh's Criterion



First object produce a diffracted object on the screen. Second object also produces a diffracted object on the screen. Rayleigh's Criteria requires the angular separation of the two be a minimum of $\theta \cong \sin\theta = \lambda/a$. If this is true, then the two objects are resolved.

Example: Photographs in newspapers are made up of a large number of closely spaced dots.
a) How far apart should these dots be such that they are not quite resolved (Rayleigh Criterion) when the printed picture is held 25 cm from your eye?
b) How does this compare with a low end laser printer with 300 x 300 dpi?

Example: Photographs in newspapers are made up of a large number of closely spaced dots.
a) How far apart should these dots be such that they are not quite resolved (Rayleigh Criterion) when the printed picture is held 25 cm from your eye?
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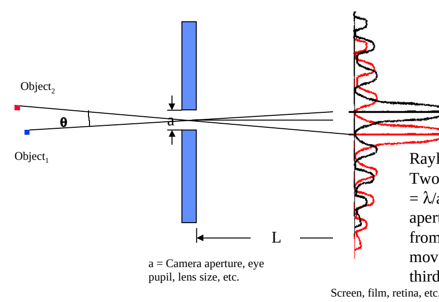
a) Rayleigh's Criteria requires the angular separation of the two be $\theta \cong \sin\theta = \lambda/a$.

λ = visible light, so λ = 600 nm.
a = aperture, your pupil. Thus, a = 3 mm.
$\sin\theta = y/25\text{cm}$, where y is the dot spacing!
$y/25\text{cm} = \lambda/a = 600\text{nm}/3\text{mm} \rightarrow y = 0.5$ mm

b) 300 dpi = 300 dots/inch

300 dpi = 300 dots/inch x 1Inch/25.4mm = 11.8 dots/mm
or 11.8 dots/mm $\rightarrow$ 0.1 mm/dot
0.1 mm/dot well within Rayleigh's Criteria, 0.5 mm in this case

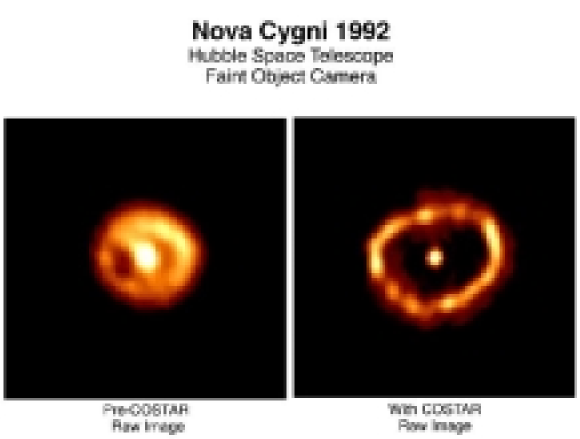
R. C. for a Circular Aperture



Rayleigh's Criteria in Two Dimensions $\theta \cong \sin\theta = \lambda/a$. But a circular aperture has contributions from a 2-D aperture moving to a screen in the third dimension!

Rayleigh's Criteria for circular aperture takes these effects into account

$$\theta \cong \sin\theta = 1.22\ \lambda/a.$$



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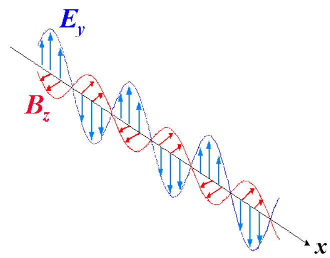
a) Rayleigh's Criteria requires the angular separation of the two be $\theta \cong \sin\theta = 1.22\,\lambda/a$.

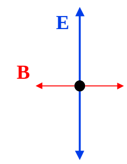
λ = visible light, so λ = 600 nm.
a = aperture, your pupil. Thus, a = 3 mm.
$\sin\theta = y/25\text{cm}$, were y is the dot spacing!
$y/25\text{cm} = 1.22\,\lambda/a = 1.22\ 600\text{nm}/3\text{mm} \rightarrow y = 0.6$ mm, not a big change!!!

Polarization

Previously we described the transverse nature of E&M waves as follows



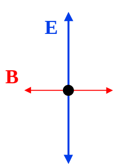
End view (coming at you)

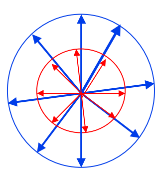


i.e. plane waves

Polarization

The E and B fields must always be perpendicular.
Unpolarized light consist of light with fields in all random planes.

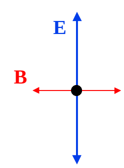




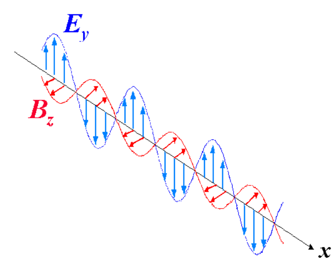
In unpolarized light, all positions (or planes) are equally probable.
Superposition yields the net field at any instant!

Polarization

Linear polarized light has the E field in one plane (and thus the B field in a plane at 90°.

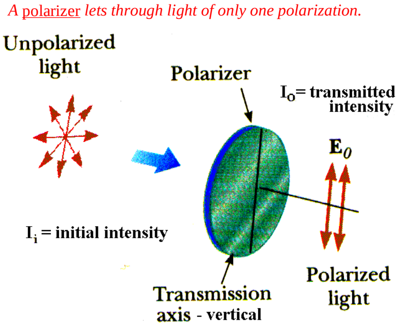


End view



Polarization - Polarizer

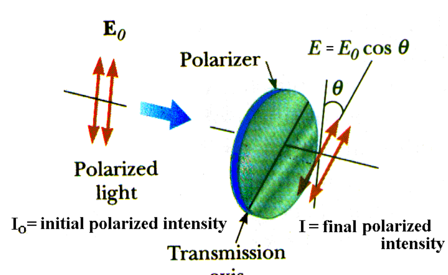
Light can be polarized by a device called a polarizer.
A polarizer lets through light of only one polarization.



$I_0 = I_i/2$, where I_i was unpolarized light

Polarization - Polarizer

Polarized light can be altered by a polarizer.



$I = \varepsilon_o \frac{E^2}{2}$

$= \varepsilon_o \frac{E_o^2 \cos^2\theta}{2}$

Law of Malus
$I = I_o \cos^2\theta$