

Geometrical Optics

Chapter 36

Images

Plane Mirrors

Spherical (concave) Mirrors

Convex Mirrors

Images by Refraction

Thin Lenses

Image Formation

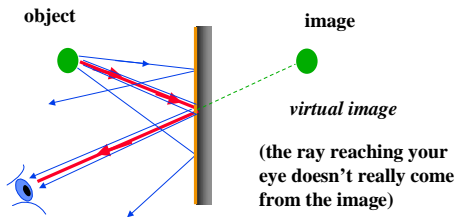
We'll stick to geometrical optics: light propagates in straight lines until its direction is changed by reflection or refraction.

When we see an **object** directly, light comes to us straight from the object.

When we use mirrors and lenses, we see light that seems to come straight from the object but actually doesn't.

Thus we see an **image**, which may have a different position, size, or shape than the actual object.

Images formed by plane mirrors



Your brain thinks the ray came from the image.

Images formed by plane mirrors

You can locate each point on the image with two rays.

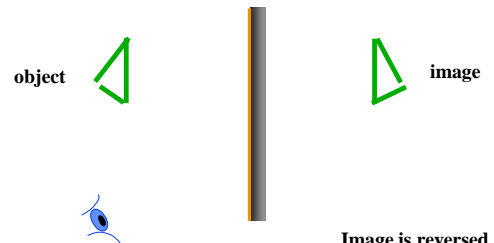


Image is reversed (front to back)

Images formed by plane mirrors

You can locate each point on the image with two rays.

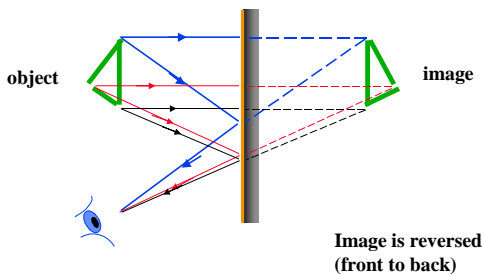
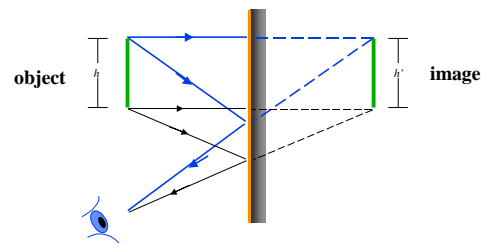


Image is reversed (front to back)

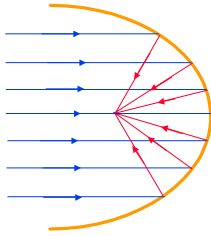
Images formed by plane mirrors

Lateral magnification, M , is given by $M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$



Parabolic Mirrors

- Shape the mirror into a parabola of rotation (In one plane it has cross section given by $y = x^2$).
- All light going into such a mirror parallel to the parabola's axis of rotation is reflected to pass through a common point - the focus.
- What about the reverse?



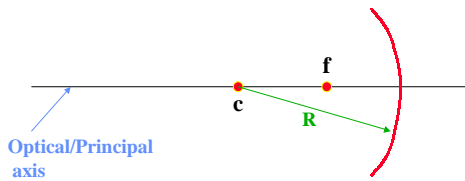
Parabolic Mirrors

- These present the concept of a focal point - the point to which the optic brings a set of parallel rays together.
- Parallel rays come from objects that are very far away.
- Parabolas are hard to make. It's much easier to make spherical optics, so that's what we'll examine next.

Spherical Mirrors

A spherical mirror has

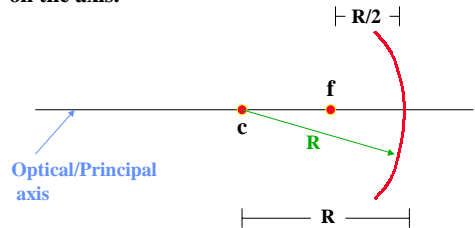
its center at 'c' and the radius of the sphere is 'R' and the focal point 'f' half way between 'c' and the mirror on the axis.



Spherical Mirrors

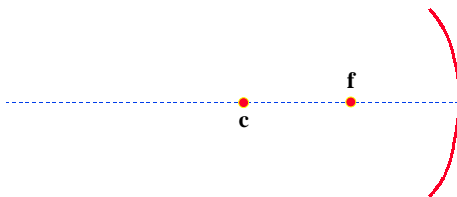
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its center at 'c' and the radius of the sphere is 'R' and the focal point 'f' half way between 'c' and the mirror on the axis.



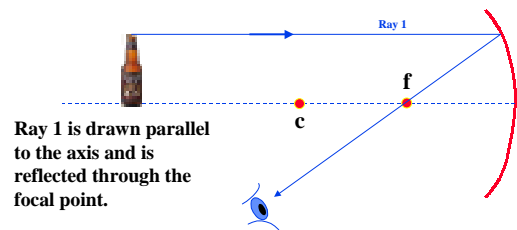
Spherical Mirrors

To analyze how a spherical mirror works we draw some special rays, apply the law of reflection where they strike the spherical surface and find out where they intersect.



Spherical Mirrors

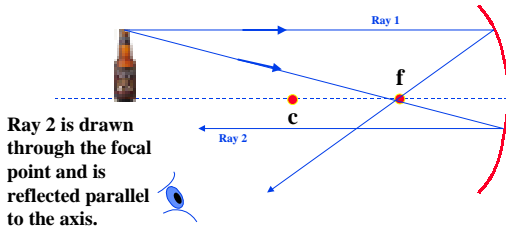
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Ray 1 is drawn parallel to the axis and is reflected through the focal point.

Spherical Mirrors

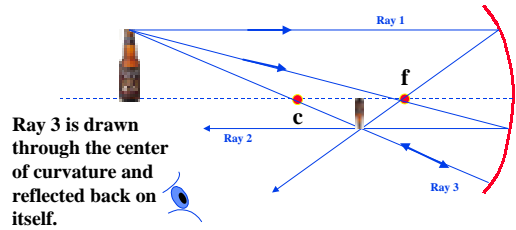
To analyze how a spherical mirror works we draw some special rays, apply the law of reflection where they strike the spherical surface and find out where they intersect.



Ray 2 is drawn through the focal point and is reflected parallel to the axis.

Spherical Mirrors

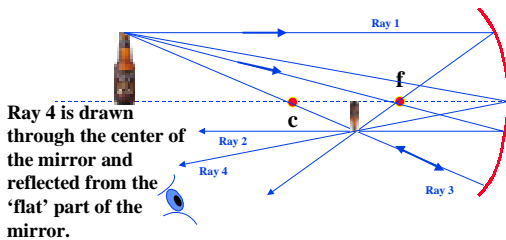
To analyze how a spherical mirror works we draw some special rays, apply the law of reflection where they strike the spherical surface and find out where they intersect.



Ray 3 is drawn through the center of curvature and reflected back on itself.

Spherical Mirrors

To analyze how a spherical mirror works we draw some special rays, apply the law of reflection where they strike the spherical surface and find out where they intersect.

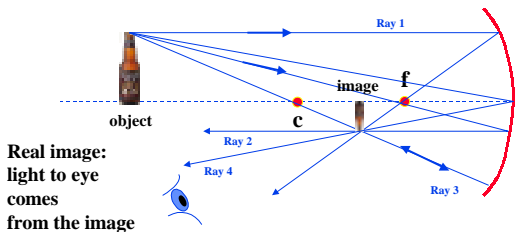


Ray 4 is drawn through the center of the mirror and reflected from the 'flat' part of the mirror.

Spherical Mirrors

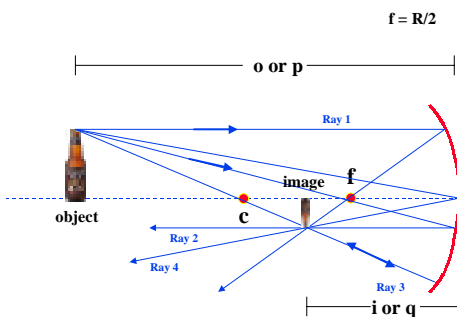
Note: every ray which leaves the tip of the object will go through the tip of the image!

All other rays leaving from other locations on the object will go through that corresponding location on the image!!! Thus, the image looks like the object to the eye.

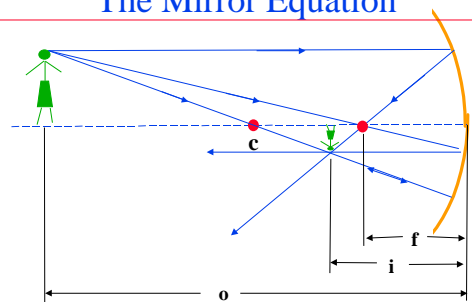


Real image: light to eye comes from the image

Spherical Mirrors

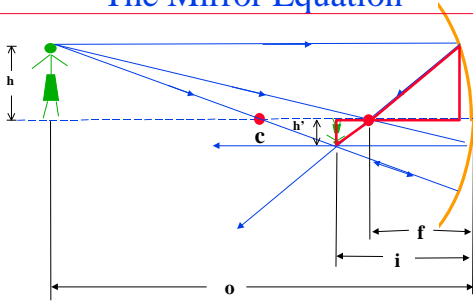


The Mirror Equation



Here $f=R/2$

The Mirror Equation

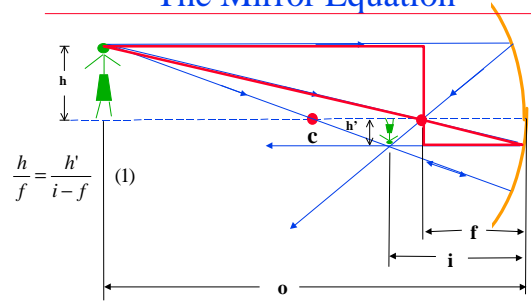


$$\frac{h}{f} = \frac{h'}{i-f} \quad (1)$$

From the tangent of the common angle between these two triangles:

Here $f=R/2$

The Mirror Equation



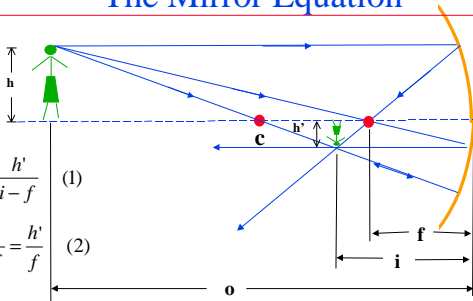
$$\frac{h}{f} = \frac{h'}{i-f} \quad (1)$$

From the tangent of the common angle between these two triangles:

$$\frac{h}{o-f} = \frac{h'}{f} \quad (2)$$

Here $f=R/2$

The Mirror Equation



$$\frac{h}{f} = \frac{h'}{i-f} \quad (1)$$

$$\frac{h}{o-f} = \frac{h'}{f} \quad (2)$$

Dividing eq'n (1) by (2) gives:

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{f}$$

Here $f=R/2$

$$\frac{h}{f} = \frac{h'}{i-f} \quad (1) \quad \frac{h}{o-f} = \frac{h'}{f} \quad (2)$$

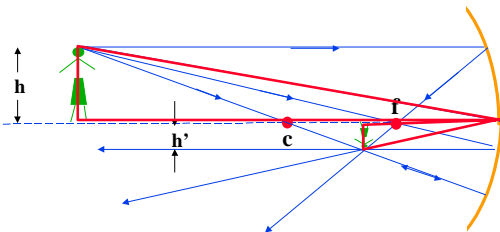
$$\frac{(2)}{(1)} = \frac{f}{o-f} = \frac{i-f}{f} \rightarrow (o-f)f \left[\frac{f}{o-f} = \frac{i-f}{f} \right]$$

$$f^2 = oi - of - if + f^2 \rightarrow 0 = oi - of - if$$

$$oi = of + if \rightarrow \frac{1}{oif} [oi = of + if]$$

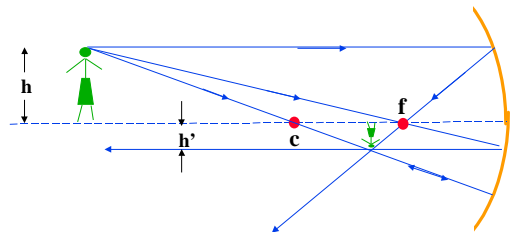
$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$

Magnification



The magnification is given by the ratio $M=h'/h$
 $\theta_i = -\theta_r$, $\tan\theta_i = h/o$ and $\tan\theta_r = h'/i$, So $M = -i/o$

Magnification



The magnification is given by the ratio $M=h'/h=-i/o$

Curved Mirrors

Simple rules: **mirror equation** $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$

focal length $f = R / 2$

magnification $M = \frac{h'}{h} = -\frac{i}{o}$

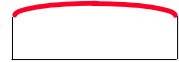
formulas are approximate - curved mirror approximates a parabolic mirror

Positive and Negative Mirrors

- You can fill a positive mirror with water.
- You can't fill a negative mirror.



positive mirror is concave

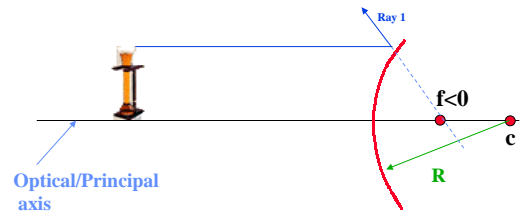


negative mirror is convex

Image With a Negative Mirror



Image With a Negative Mirror



Ray 1 is drawn parallel to the axis and is reflected through the focal point. Note the change for negative or concave mirrors.

Image With a Negative Mirror

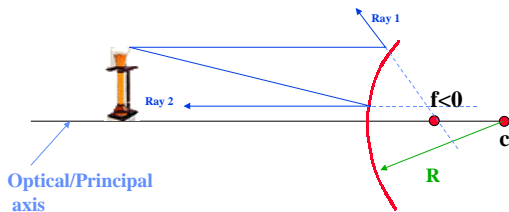
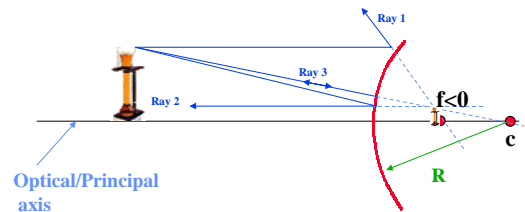


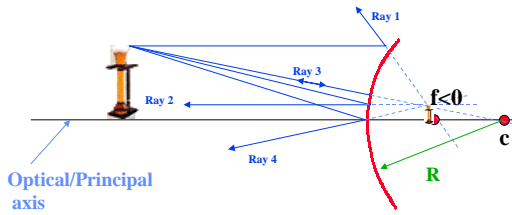
Image With a Negative Mirror



Ray 2 is drawn through the focal point and is reflected parallel to the axis. Note the change for negative or concave mirrors.

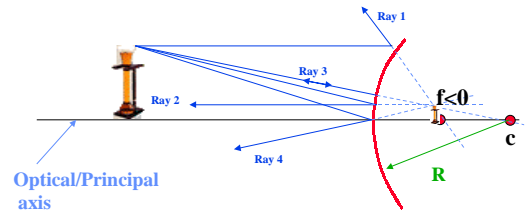
Ray 3 is drawn through the center of curvature and reflected back on itself. Note the change for negative or concave mirrors.

Image With a Negative Mirror



Ray 4 is drawn through the center of the mirror and reflected from the 'flat' part of the mirror. Note the change for negative or concave mirrors (?).

Image With a Negative Mirror



Here the image is virtual, apparently positioned behind the mirror. Can still use the mirror equation (with negative distances).

Sign Convention

- Objects and real images have positive distances measured along the axis of the mirror.
- Virtual images have negative distances.
- Mirrors that can make real images have positive focal lengths and ones that can't have negative focal lengths.

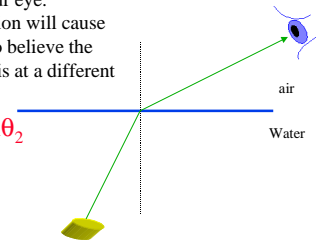
Images Formed by Refraction

Consider looking from air into water

Rays from objects leaving the water will be refracted into air before reaching your eye.

This refraction will cause you and your eye to believe the object in the water is at a different location!

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Images Formed by Refraction

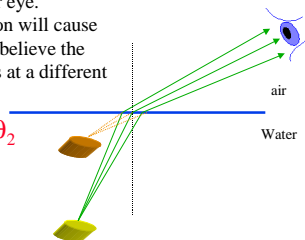
Consider looking from air into water

Our eye detects an image as shown.

Rays from objects leaving the water will be refracted into air before reaching your eye.

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$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



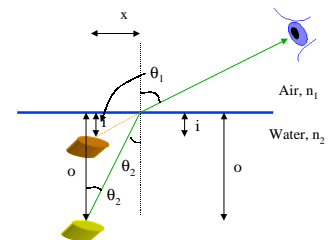
Images Formed by Refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

and

$$i \tan \theta_1 = o \tan \theta_2 = x$$

Our eye detects an image as shown.



Images Formed by Refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2, \sin \theta_2 = n_1 \sin \theta_1 / n_2$$

and

$$i \tan \theta_1 = o \tan \theta_2$$

so

$$i \sin \theta_1 \cong o \sin \theta_2 \text{ for small angles}$$

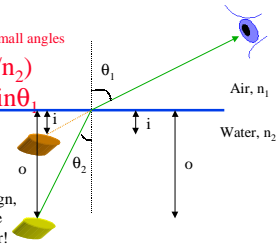
$$i \sin \theta_1 \cong o (n_1 \sin \theta_1 / n_2)$$

$$i \sin \theta_1 \cong (n_1 / n_2) o \sin \theta_1$$

or

$$i \cong (n_1 / n_2) o$$

Note: your text has a minus sign,
 $i = -(n_1/n_2)o$, because they have
 assumed the image is in the air!



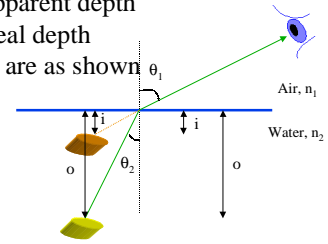
Images Formed by Refraction

Thus the apparent depth of an object is given by $i = -(n_1/n_2)o$, where

i is the apparent depth

o is the real depth

n_1 and n_2 are as shown

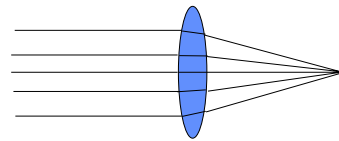


Lenses

- First the concept of the optical path length:
 - We have a physical path length which is just what you measure with your ruler.
 - Multiply that by the index of refraction and you have the optical path length.
- A lens is an optical device in which the optical path length is varied so that images can be formed.

The Simple Lens - Simply

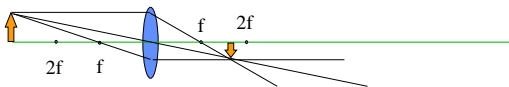
A simple lens is an optical device which takes parallel light rays and focuses them to a point.



Snell's Law applied at each surface will show where the light comes to a focus.

Some Simple Ray Traces

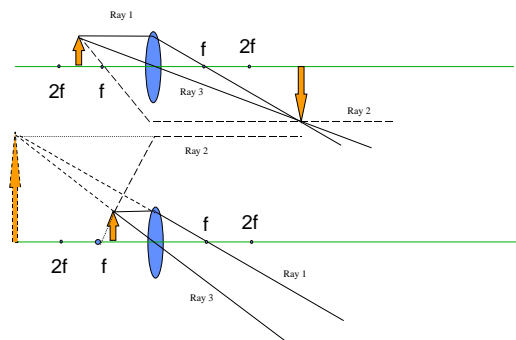
A simple positive lens is one which can produce a real image.



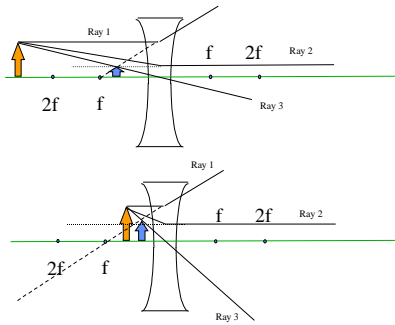
Each point in the image can be located using two rays. Three key rays are particularly simple:

1. A ray which leaves the object parallel to the axis is refracted to pass through the focal point, and
2. A ray passing through the focal point is refracted to end up parallel to the axis.
3. A ray which passes through the lens's center is undeflected.

Some Simple Ray Traces (cont'd)



Some Simple Ray Traces (cont'd)



What happens if the lower half of the lens were cut off?

Types of Lenses

Positive lenses are ones which can form real images. These are convex, and converging.

You can not hold water in a positive lens.

Negative lenses are ones which can not form real images. These are concave, and diverging.

You can hold water in a negative lens.

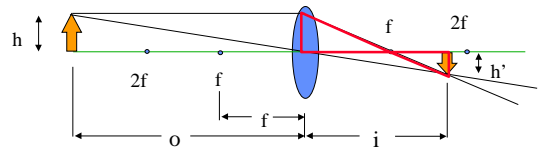
For now we will only deal with thin lenses.

Lenses where the physical thickness can be ignored.

The Sign Convention

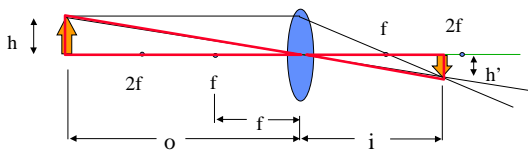
- Objects and real images have positive distances; Virtual images have negative distances.
- Converging lenses (positive lenses) have positive focal lengths and diverging lenses (negative lenses) have negative focal lengths.
- All distances are measured along the axis of the lens.
- “Power”, P , of a lens = $1/f$ measured in **Diopters**
e.g., if $f = 25 \text{ cm} = 0.25 \text{ m}$, $P = 1/0.25$
 $= 4 \text{ diopters}$

Derivation of the Lens Equation



$h'/(i - f) = h/f$ by having similar triangles.

Derivation of the Lens Equation



$h'/(i - f) = h/f$ by having similar triangles and
 $h'/i = h/o$ by similar triangles. Divide these
two eq'ns by each other to eliminate h and h'
to get

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \text{The thin lens eq'n}$$

$$\frac{h'}{i} = \frac{h}{o} \quad (1)$$

$$\frac{h'}{i - f} = \frac{h}{f} \quad (2)$$

$$\frac{(1)}{(2)} = \frac{\frac{h'}{i}}{\frac{h'}{i - f}} = \frac{\frac{h}{o}}{\frac{h}{f}} = \frac{1}{i} = \frac{1}{i - f} = \frac{i - f}{i} = \frac{f}{o}$$

$$\frac{1}{i} - \frac{f}{i} = \frac{f}{o}$$

$$\frac{1}{f} - \frac{1}{i} = \frac{1}{o}$$

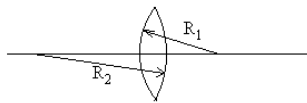
$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

About the Thin Lens Formula

$$1/i + 1/o = 1/f$$

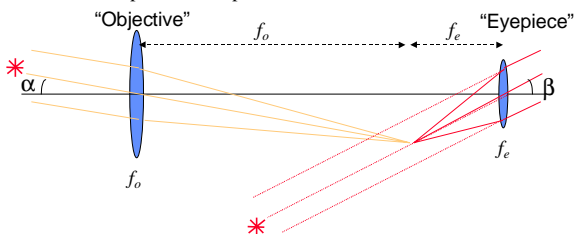
- When $o = f$, $i = \text{infinity}$
- When $o = 2f$, $i = 2f$ and the magnification is 1.
- When $f > o > 0$, i is negative
 - This means that the image is virtual, and so it is on the same side of the lens as the object.
- If $f < 0$, i is always negative
 - A negative lens can not produce a real image. It always produces a virtual image.

$$(n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$



Multiple Lens

We have examined single lens, both converging and diverging types. But, image forming instruments commonly have more than one lens! For example, a telescope.



The Lensmaker's Formula

We can calculate the focal length f given the shape of the lens, and using Snell's law.

Keeping the angles of incidence small so that $\sin\theta = \theta$ you can show by geometry that

$$(n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$

where n is the index of the lenses glass, R_1 is the radius of curvature of the front surface, and R_2 is the radius of curvature of the back surface.

Other Comments About Lenses

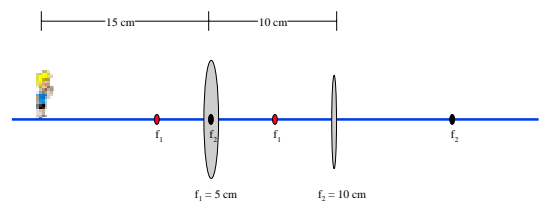
- The radii of curvature can be both positive and negative and one lens can have one of each.

$$(n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$

$$\frac{1}{f_o} = \frac{1}{f_1} + \frac{1}{f_2}$$

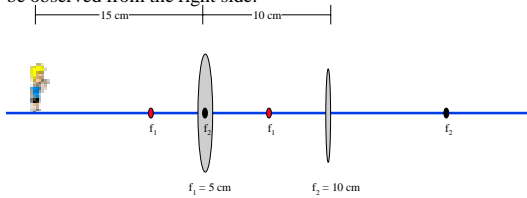
Example: Two Lens

Consider two converging lenses and an image, where will the image be formed as viewed through the second lens and what will the magnification be?



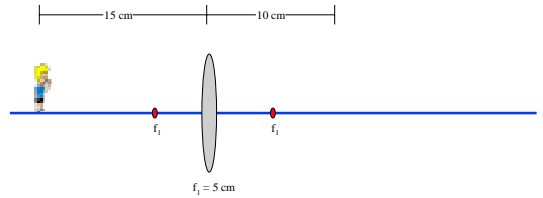
Example: Two Lens

The image the first lens produces become the object that the second lens sees. Thus the image of the first lens becomes the object of the second. The second produces an image which will be observed from the right side.



Example: Two Lens

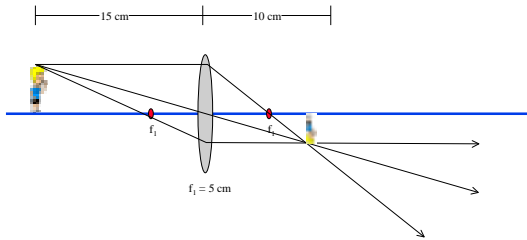
First lens first!



Example: Two Lens

First lens first!

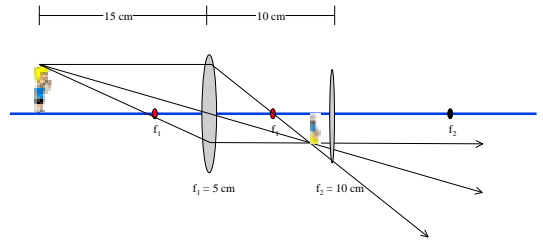
$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o} \right)^{-1} = \left(\frac{1}{5\text{cm}} - \frac{1}{15\text{cm}} \right)^{-1} = 7.5\text{cm}$$



Example: Two Lens

Now do the second!

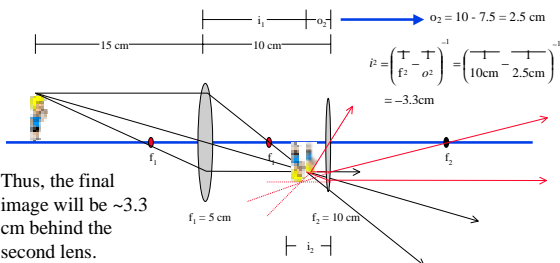
$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o} \right)^{-1} = \left(\frac{1}{10\text{cm}} - \frac{1}{15\text{cm}} \right)^{-1} = 7.5\text{cm}$$



Example: Two Lens

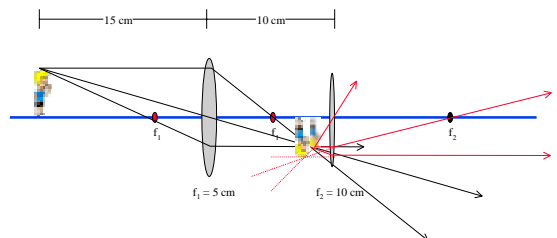
Now do the second!

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o} \right)^{-1} = \left(\frac{1}{5\text{cm}} - \frac{1}{15\text{cm}} \right)^{-1} = 7.5\text{cm}$$



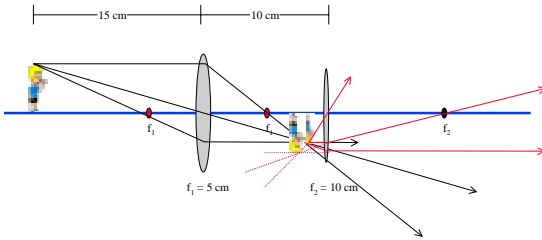
Example: Two Lens

Last, the total magnification.



Example: Two Lens

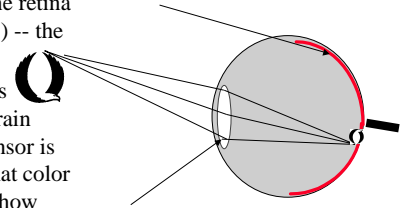
Last, the total magnification. $M = M_1 M_2 = \left(-\frac{i_1}{o_1} \right) \left(-\frac{i_2}{o_2} \right) = \left(-\frac{7.5}{15} \right) \left(-\frac{-3.3}{2.5} \right)$
 $M = -0.67$



The Eye: A simple imager

- A simple lens focuses the light onto the retina (rods and cones) -- the photosensor
- The retina sends signals to the brain about which sensor is illuminated, what color is the light and how much of it there is.
- The brain interprets.

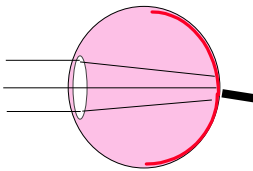
The retina senses the light.



Your eye's lens is a simple refracting lens which you can deform to change focus.

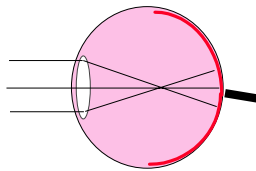
The Eye: Near & Farsightedness

Far-sighted:



Lens too weak
- correct with a converging lens

Near-sighted:

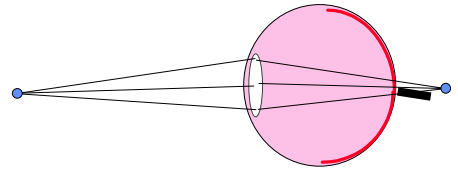


Lens too strong
- correct with a diverging lens

The Eye: Farsightedness

Near objects can not be focused on the retina. These objects are focused behind the eye.

Far-sighted:



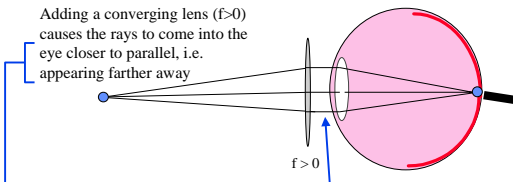
Lens too weak
- correct with a converging lens

The Eye: Farsightedness

Near objects can not be focused on the retina. These objects are focused behind the eye.

Far-sighted:

Adding a converging lens ($f > 0$) causes the rays to come into the eye closer to parallel, i.e. appearing farther away

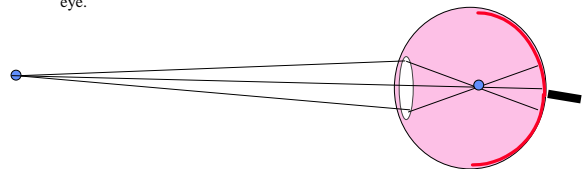


Lens too weak
- correct with a converging lens

The Eye: Nearsightedness

Far objects can not be focused on the retina. These objects are focused before the back of the eye.

Near-sighted:

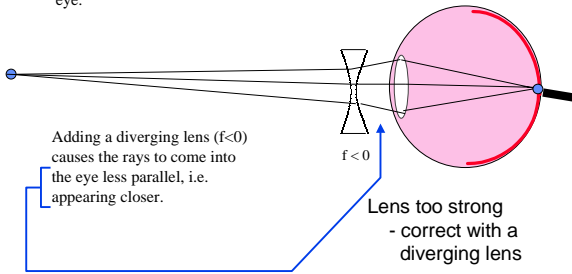


Lens too strong
- correct with a diverging lens

The Eye: Nearsightedness

Far objects can not be focused on the retina. These objects are focused before the back of the eye.

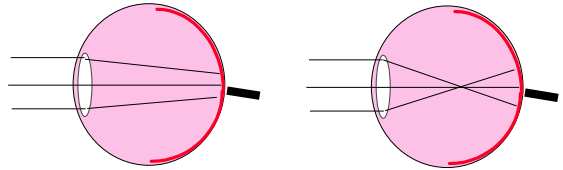
Near-sighted:



The Eye: Near & Farsightedness

Far-sighted:

Near-sighted:



Optometrists and ophthalmologists prescribe corrective lenses measured in **diopters**. The power of a lens in diopters is $1/f$ where f is measured in meters

Example: Prescribing Glasses

A nearsighted person can't clearly focus objects more than 1.8 meters away. What power of lens should be prescribed to correct the nearsightedness?

Example: Prescribing Glasses

A nearsighted person can't clearly focus objects more than 1.8 meters away. What power of lens should be prescribed to correct the nearsightedness?

Sol'n

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

Want to be able to see objects far away, $o \rightarrow \infty$, but we can only see near objects, $i \rightarrow 1.8$ m. Optically, $i \rightarrow -1.8$ m. So

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{1}{\infty} + \frac{1}{-1.8m} \cong -0.56m^{-1}$$

Power in diopters = $1/f$ in meters

$P = -0.56$ diopters.