

# Chapter 35

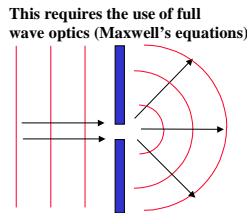
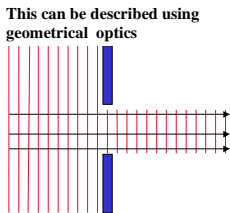
## Nature of Light Laws of Geometrical Optics

### Geometrical Optics

- Optics is the study of the behavior of light (not necessarily visible light).
- This can be described by Maxwell's equations (except in very small systems, e.g., atoms).
- But sometimes the light travels in straight lines called rays, and its wave nature can be ignored.
- This is the realm of geometrical optics.
- In the next two chapters the case of geometrical optics is developed. The wave properties of light show up in phenomena such as interference and diffraction (physical optics).

### Geometrical Optics

Light can be described using geometrical optics as long as the objects with which it interacts are much larger than the wavelength of the light.



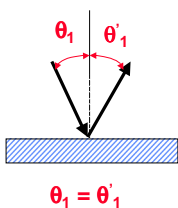
### Reflection and Transmission

Some materials reflect light. For example, metals reflect light because an incident oscillating light beam causes the metal's nearly free electrons to oscillate, setting up another electromagnetic wave.

Opaque materials absorb light (by, say, moving electrons into higher atomic orbitals).

Transparent materials are usually insulators whose electrons are bound to atoms, and which would require more energy to move to higher orbitals than in materials which are opaque.

### Reflection

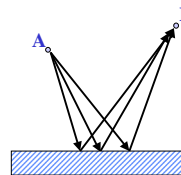


This is called "specular" reflection

#### The Law of Reflection:

Light reflected from a surface stays in the plane formed by the incident ray and the surface normal; and the angle of reflection equals the angle of incidence (measured to the normal)

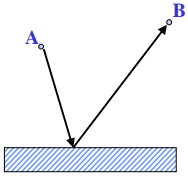
### Reflection



#### Fermat's Principle:

Light traveling from Point A to Point B will take the path which requires the least time.

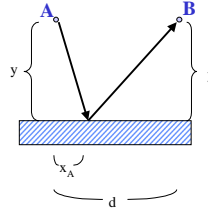
## Reflection



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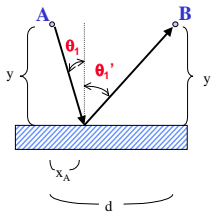
## Reflection



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$$\text{time} = t_{AB} = \frac{\text{dist.}}{\text{speed}} = \frac{\sqrt{y^2 + x_A^2}}{c} + \frac{\sqrt{y^2 + (d-x_A)^2}}{c}$$

$$\frac{\partial t_{AB}}{\partial x_A} = \frac{1}{c} \left( \frac{2x_A}{\sqrt{y^2 + x_A^2}} - \frac{2(d-x_A)}{\sqrt{y^2 + (d-x_A)^2}} \right) = 0$$

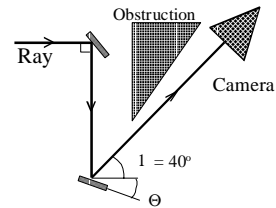
$$0 = \frac{2x_A}{\sqrt{y^2 + x_A^2}} - \frac{2(d-x_A)}{\sqrt{y^2 + (d-x_A)^2}}$$

$$\frac{x_A}{\sqrt{y^2 + x_A^2}} = \frac{d-x_A}{\sqrt{y^2 + (d-x_A)^2}}$$

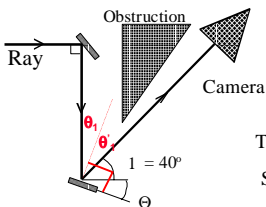
$$\sin \theta_1 = \frac{x_A}{\sqrt{y^2 + x_A^2}} \quad \text{and} \quad \sin \theta_1' = \frac{d-x_A}{\sqrt{y^2 + (d-x_A)^2}}$$

thus,  $\sin \theta_1 = \sin \theta_1'$  or  $\theta_1 = \theta_1'$

**Example:** Find angle  $\Psi$ , such that the camera can observe the ray around the obstruction.

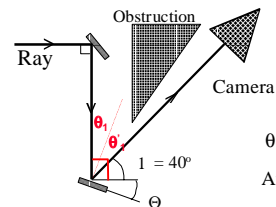


**Example:** Find angle  $\Psi$ , such that the camera can observe the ray around the obstruction.



Thus,  $\theta_1' + \Theta + \Psi = 90^\circ$   
Still need  $\theta_1'$  to get  $\Psi$

**Example:** Find angle  $\Psi$ , such that the camera can observe the ray around the obstruction.



$$\theta_1 + \theta_1' + \Theta = 90^\circ$$

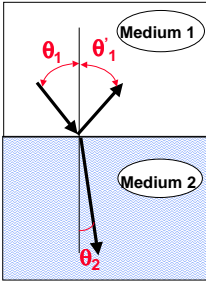
And,  $\theta_1 = \theta_1'$

$$\theta_1' + \theta_1' + \Theta = 90^\circ$$

$$\theta_1' = 25^\circ$$

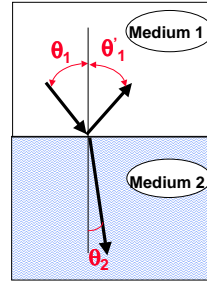
$$\text{From } \theta_1' + \Theta + \Psi = 90^\circ \rightarrow \Psi = 25^\circ$$

## Refraction



More generally, when light passes from one transparent medium to another, part is reflected and part is transmitted. The reflected ray obeys  $\theta_1 = \theta'_1$ .

## Refraction



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The transmitted ray obeys

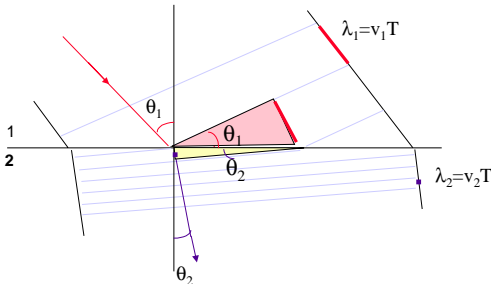
**Snell's Law of Refraction:**

It stays in the plane, and the angles are related by

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

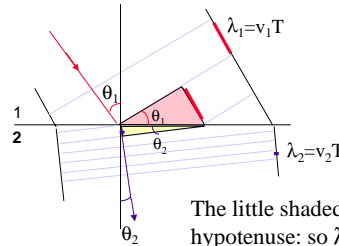
Here  $n$  is the "index of refraction" of a medium.

## Refraction



The period  $T$  doesn't change, but the speed of light can be different in different materials. Then the wavelengths  $\lambda_1$  and  $\lambda_2$  are unequal. This also gives rise to refraction.

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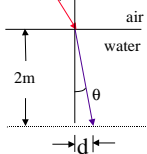
The little shaded triangles have the same hypotenuse: so  $\lambda_1 / \sin \theta_1 = \lambda_2 / \sin \theta_2$ , or  $v_1 / \sin \theta_1 = v_2 / \sin \theta_2$

Define the index of refraction:  $n = c/v$ . Then Snell's law is:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

## Example: air-water interface

If you shine a light at an incident angle of  $40^\circ$  onto the surface of a pool 2m deep, where does the beam hit the bottom?

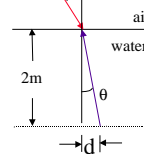
Air:  $n=1.00$     Water:  $n=1.33$



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Air:  $n=1.00$     Water:  $n=1.33$



$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ (1.00) \sin 40 &= (1.33) \sin \theta \\ \sin \theta &= \sin 40 / 1.33 \quad \text{so } \theta = 28.9^\circ \end{aligned}$$

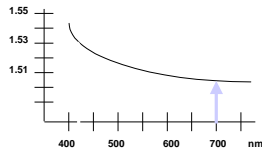
Then  $d/2 = \tan 28.9^\circ$  which gives  $d = 1.1$  m.

Turn this around: if you shine a light from the bottom at this position it will look like it's coming from further right.

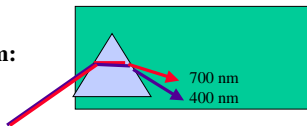
## Dispersion

The index of refraction depends on frequency or wavelength:  $n = n(\lambda)$

Typically many optical materials, (glass, quartz) have decreasing  $n$  with increasing wavelength in the visible region of spectrum

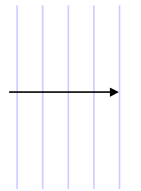


Dispersion by a prism:



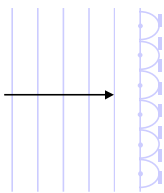
## Huygen's principle

Huygen first explained this in 1678 by proposing that all planar wave fronts are made up of lots of spherical wave fronts..



## Huygen's principle

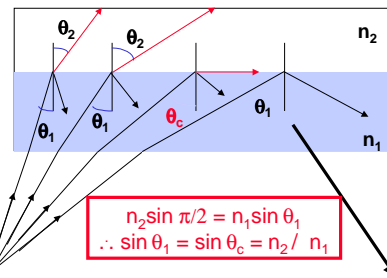
Huygen first explained this in 1678 by proposing that all planar wave fronts are made up of lots of spherical wave fronts.



That is, you see how light propagates by breaking a wave front into little bits, and then draw a spherical wave emanating outward from each little bit. You then can find the leading edge a little later simply by summing all these little "wavelets"

It is possible to explain reflection and refraction this way too.

## Total Internal Reflection



Some is refracted and some is reflected

Total internal reflection: none is refracted

$$n_2 \sin \pi/2 = n_1 \sin \theta_1$$

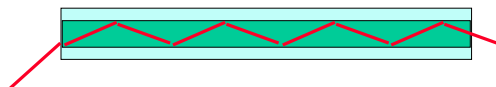
$$\therefore \sin \theta_1 = \sin \theta_c = n_2 / n_1$$

## Total Internal Reflection

- Suppose the light goes from medium 1 to 2 and that  $n_2 < n_1$  (for example, from water to air).
- Snell's law gives  $\sin \theta_2 = (n_1 / n_2) \sin \theta_1$ .
- Since  $\sin \theta_2 \leq 1$  there must be a maximum value of  $\theta_1$ .
- At angles bigger than this "critical angle", the beam is totally reflected.
- The critical angle is when  $\theta_2 = \pi/2$ , which gives  $\theta_c = \sin^{-1}(n_2/n_1)$ .

## Example: Fiber Optics

An optical fiber consists of a core with index  $n_1$  surrounded by a cladding with index  $n_2$ . If  $n_1 > n_2$ , then light can be confined by total internal reflection, even if the fiber is bent and twisted.



Exercise: For  $n_1 = 1.7$  and  $n_2 = 1.6$  find the minimum angle of incidence for guiding in the fiber.

Answer:  $\theta_c = \sin^{-1}(1.6/1.7) = 70^\circ$ . (Need to graze at  $< 20^\circ$ )