Chapter 34

The Laws of Electromagnetism
Maxwell's Equations
Electromagnetic Radiation
Laws of Geometrical Optics

Maxwell's Equations

$$\oiint_{\substack{closed\\surface}} \vec{E} \bullet d\vec{A} = \frac{q_{inclose}}{\varepsilon_o}$$ Gauss's Law for Electricity

$$\oiint_{\substack{closed\\surface}} \vec{B} \bullet d\vec{A} = 0$$ Gauss's Law for Magnetism

$$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt} = EMF$$ Faraday's Law

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{s} = \mu_o I_{inclosed} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$ Ampere's Law as Modified by Maxwell

Maxwell's Equations - Static

$$\oiint_{\substack{closed\\surface}} \vec{E} \bullet d\vec{A} = \frac{q_{inclose}}{\varepsilon_o}$$ Gauss's Law for Electricity

$$\oiint_{\substack{closed\\surface}} \vec{B} \bullet d\vec{A} = 0$$ Gauss's Law for Magnetism

Symmetry, but we have no magnetic monopoles.

If we had magnetic monopoles, then $\oiint_{\substack{closed\\surface}} \vec{B} \bullet d\vec{A} = \mu_o \rho_{inclose}$

where ρ is the monopole.

But, we have not detected any magnetic monopoles, so $\oiint_{\substack{closed\\surface}} \vec{B} \bullet d\vec{A} = 0$

Maxwell's Equations - Dynamic

$$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$$ Faraday's Law

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{s} = \mu_o I_{inclosed} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$ Ampere's Law as Modified by Maxwell

Again we have symmetry in the E and B field, but we lack magnetic monopoles.

If we had monopoles Faraday's Law would become

$$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{s} = -\frac{I_\rho}{\varepsilon_o} - \frac{d\Phi_B}{dt}$$

Where $I_\rho = d\rho/dt$, i.e. the flow of magnetic monopoles.

Maxwell's Equations - Coupling

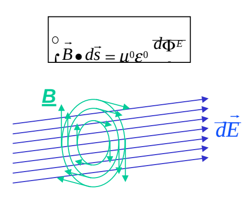
Consider Maxwell's dynamic equations in vacuum with no free charges or currents.

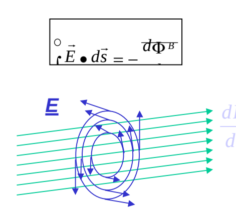
$$\oint_{\substack{closed\\loop}} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B} \bullet d\vec{A} \qquad (1)$$

$$\oint_{\substack{closed\\loop}} \vec{B} \bullet d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}, \quad \Phi_E = \int \vec{E} \bullet d\vec{A} \qquad (2)$$

How would a time changing **B** field effect an **E** field in vacuum (1) and how would a time changing **E** field effect an **B** field in vacuum (2)?

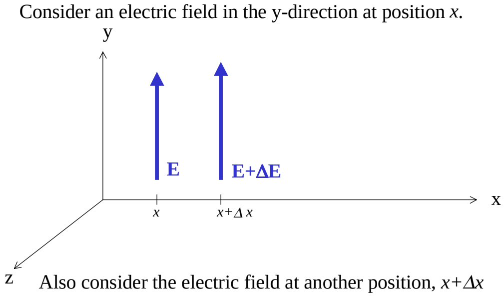
Maxwell's Equations - Coupling





Maxwell's Equations - Coupling

Consider an electric field in the y-direction at position *x*.

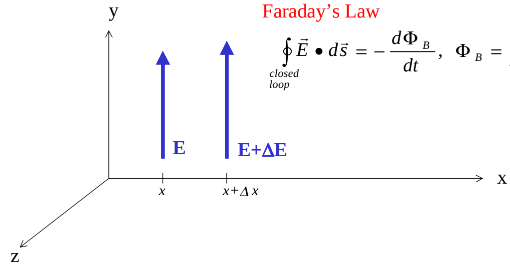


Also consider the electric field at another position, *x+Δx*

Assume the electric field only depends on the x-position!
And that the electric field fills some space.

Maxwell's Equations - Coupling

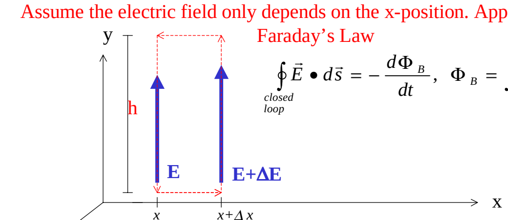
Assume the electric field only depends on the x-position. Apply, Faraday's Law



$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B}\bullet d\vec{l}$$

Maxwell's Equations - Coupling

Assume the electric field only depends on the x-position. Apply, Faraday's Law



$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B}\bullet d\vec{l}$$

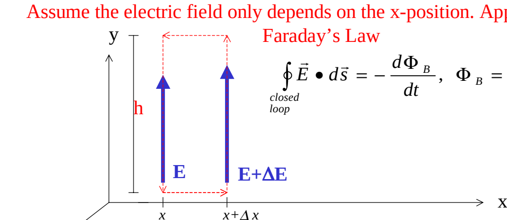
From

$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = (E+\Delta E)h + \vec{E}\cos(90)\Delta x - Eh - \vec{E}\cos(90)\Delta x$$

$$= (E+\Delta E)h + 0 - Eh - 0 = \Delta Eh$$

Maxwell's Equations - Coupling

Assume the electric field only depends on the x-position. Apply, Faraday's Law



$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B}\bullet d\vec{l}$$

From

$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = \Delta Eh$$

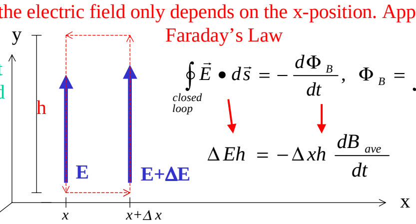
and

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(B_{ave}\Delta xh) = \Delta xh\frac{dB_{ave}}{dt}$$

Maxwell's Equations - Coupling

Assume the electric field only depends on the x-position. Apply, Faraday's Law

There must be a B field inside the loop!



$$\oint_{\substack{closed\\loop}} \vec{E}\bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B}\bullet d\vec{l}$$

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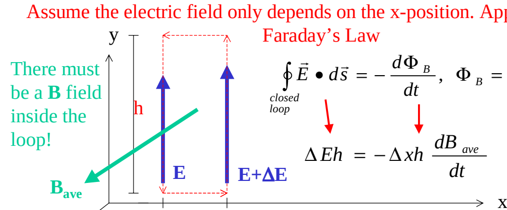
Maxwell's Equations - Coupling

Assume the electric field only depends on the x-position. Apply, Faraday's Law

There must be a **B** field inside the loop!



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Maxwell's Equations - Coupling

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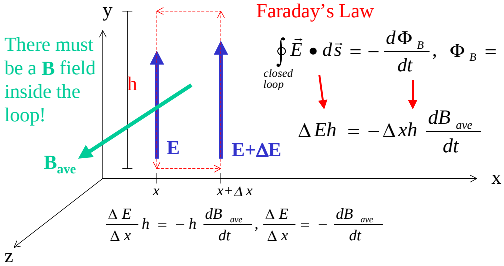


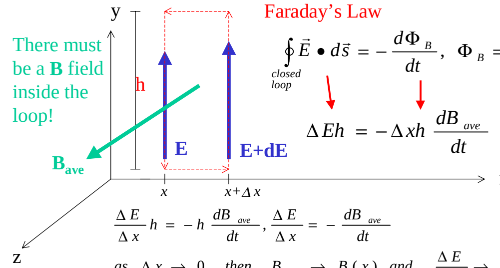
$$\oint_{closed\ loop} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B} \bullet d\vec{l}$$

$$\Delta E h = -\Delta x h \frac{dB_{ave}}{dt}$$

$$\frac{\Delta E}{\Delta x} h = -h \frac{dB_{ave}}{dt}, \quad \frac{\Delta E}{\Delta x} = -\frac{dB_{ave}}{dt}$$

Maxwell's Equations - Coupling

Assume the electric field only depends on the x-position. Apply, Faraday's Law



$$\oint_{closed\ loop} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}, \quad \Phi_B = \int \vec{B} \bullet d\vec{l}$$

$$\Delta E h = -\Delta x h \frac{dB_{ave}}{dt}$$

$$\frac{\Delta E}{\Delta x} h = -h \frac{dB_{ave}}{dt}, \quad \frac{\Delta E}{\Delta x} = -\frac{dB_{ave}}{dt}$$

$$as \ \ \Delta x \to 0, \ then \ \ B_{ave} \to B(x) \ \ and \ \ \frac{\Delta E}{\Delta x} \to \frac{dE}{dx}$$

$$Thus, \ \frac{dE}{dx} = -\frac{dB}{dt} \ \ from \ \ Faraday's \ Law!$$

Maxwell's Equations - Coupling

Faraday's law predicts that as the electric field changes in space, this will induce a changing perpendicular magnetic field at that location.

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

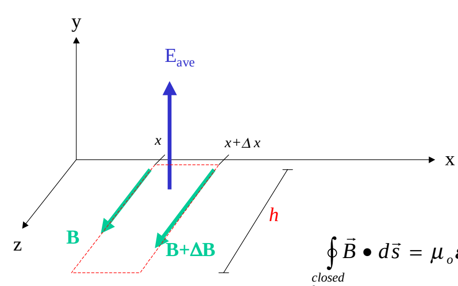
More generally,

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Where we have assumed:
E = E(x, t)**j** and
B = B(x, t)**k**.
This assumption will form *plane waves*, but in the most general case we need not make this assumption.

Maxwell's Equations - Coupling

Apply Ampere's Law (in vacuum) in a similar way. What will be the relation between E and B?



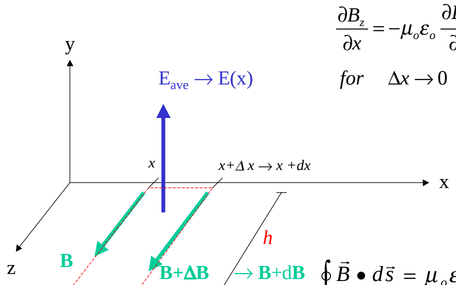
$$\oint_{closed\ loop} \vec{B} \bullet d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

Maxwell's Equations - Coupling

From Ampere's Law the relation between E and B is

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

$$for \quad \Delta x \to 0$$



$$\oint_{closed\ loop} \vec{B} \bullet d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

Maxwell's Equations - Coupling

Thus from Maxwell's dynamic equations in vacuum the perpendicular E and B fields couple as

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Maxwell's Equations - E&M Waves

Further manipulation

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \rightarrow \frac{\partial}{\partial t} \rightarrow \frac{\partial^2 B_z}{\partial t\ \partial x} = -\mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \rightarrow \frac{\partial}{\partial x} \rightarrow \frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x\ \partial t}$$

$$and\ \frac{\partial^2 B_z}{\partial x\ \partial t} = \frac{\partial^2 B_z}{\partial t\ \partial x}\ so$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

The Wave Equation for the E field.

Maxwell's Equations - E&M Waves

Further manipulation working towards the B field

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \rightarrow \frac{\partial^2 B_z}{\partial x^2} = -\mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial x\ \partial t}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \rightarrow \frac{\partial^2 E_y}{\partial t\ \partial x} = -\frac{\partial^2 B_z}{\partial t^2}$$

$$and\ \frac{\partial^2 E_y}{\partial x\ \partial t} = \frac{\partial^2 E_y}{\partial t\ \partial x}\ so$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2}$$

The Wave Equation for the B field.

Maxwell's Equations - E&M Waves

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2}$$

Wave equations!

Maxwell's Equations - E&M Waves

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

Wave equations!

$$Assume,\ \vec{E} = E_{\max} \cos(kx - \omega t)\,\hat{j}$$

$k = 2\pi/\lambda$ and $\omega = 2\pi$ f

$$Then,\ \frac{\partial^2 E_y}{\partial x^2} = -E_{\max} \cos(kx - \omega t) k^2$$

$$and\ \frac{\partial^2 E_y}{\partial t^2} = -E_{\max} \cos(kx - \omega t) \omega^2$$

$$so,\ -E_{\max} \cos(kx - \omega t) k^2 = -\mu_o \varepsilon_o E_{\max} \cos(kx - \omega t) \omega^2$$

Or $k^2 = \mu_o \varepsilon_o \omega^2$

Maxwell's Equations - E&M Waves

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2}$$

Similarly for the B field!
But, NOTE

$$Assume,\ \vec{B} = B_{\max} \cos(kx - \omega t)\hat{k},\ \hat{k}(unit\ vector) \neq k(wave\ no.)$$

$$Then,\ \frac{\partial^2 B_z}{\partial x^2} = -B_{\max} \cos(kx - \omega t) k^2$$

$$and\ \frac{\partial^2 B_z}{\partial t^2} = -B_{\max} \cos(kx - \omega t) \omega^2$$

$$so,\ -B_{\max} \cos(kx - \omega t) k^2 = -\mu_o \varepsilon_o B_{\max} \cos(kx - \omega t) \omega^2$$

Or $k^2 = \mu_o \varepsilon_o \omega^2$

Maxwell's Equations - E&M Waves

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2}$$

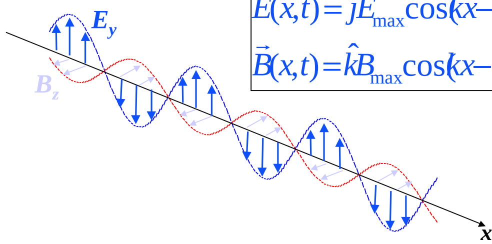
The Waves (sines and/or cosines) are a solution to this Differential Equation (uniqueness theorem)!

$$\vec{E} = E_{\max} \cos(kx - \omega t)\,\hat{j}$$

and

$$\vec{B} = B_{\max} \cos(kx - \omega t)\,\hat{k}$$

Plane electromagnetic waves



$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Maxwell's Equations - E&M Waves

Back to $k^2 = \mu_o \varepsilon_o \omega^2$
Recall $k = 2\pi/\lambda$, where λ is the wavelength
and $\omega = 2\pi f$, where f is the wave's frequency.

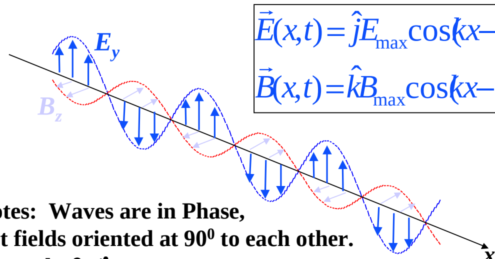
$$\left(\frac{2\pi}{\lambda}\right)^2 = \mu_o \varepsilon_o (2\pi f)^2$$

$$speed = \lambda f = \sqrt{\frac{1}{\mu_o \varepsilon_o}} = c, \text{ the speed of light!}$$

Where we have assumed the E field in the y-direction, the B field in the z-direction, and the propagation of the wave in the x-direction!

Plane electromagnetic waves

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$



Notes: Waves are in Phase,
but fields oriented at 90^0 to each other.
$k = 2\pi/\lambda$.
Speed of wave is $c = \omega/k$ (= $f\lambda$)
$$c = 1/\sqrt{\varepsilon_0 \mu_0} = 3\times 10^8 m/s$$

Maxwell's Equations - E&M Waves

Back to Coupling
Recall we coupled the electric and magnetic fields. We then de-coupled them to get the wave eq'n!
Go back to Coupling

From Maxwell's dynamic equations in vacuum the perpendicular E and B fields couple as

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \qquad \vec{E} = E_{max}\cos(kx - \omega t)\hat{j}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \vec{B} = B_{max}\cos(kx - \omega t)\hat{k}$$

For our goal now, these eq'ns are redundant. Use only the second one.

Maxwell's Equations - E&M Waves

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \vec{E} = E_{max}\cos(kx - \omega t)\hat{j}$$

$$\frac{\partial E_y}{\partial x} = -E_{max} k \sin(kx - \omega t) \qquad \vec{B} = B_{max}\cos(kx - \omega t)\hat{k}$$

$$-\frac{\partial B_z}{\partial t} = -B_{max}\omega \sin(kx - \omega t)$$

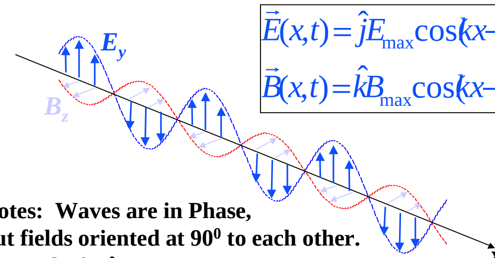
So, $-E_{max}k\sin(kx-\omega t) = -B_{max}\,\omega\sin(kx-\omega t)$

Reduces to, $E_{max}k = B_{max}\,\omega$, $\omega/k = \lambda f = c$ = speed of light

Thus, $\boxed{E_{max} = cB_{max}}$ ***Coupling!***

Plane electromagnetic waves

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$



Notes: Waves are in Phase,
but fields oriented at 90^0 to each other.
$k = 2\pi/\lambda$.
Speed of wave is $c = \omega/k$ (= $f\lambda$)
At all times E=cB.

Maxwell's Equations - E&M Waves

Summary,

$$\frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t} \quad and \quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad Coupling\ eq'ns$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2} \quad and \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 B_z}{\partial t^2} \qquad Wave\ eq'ns$$

$$\vec{E} = E_{\max} \cos(kx - \omega t)\,\hat{j}$$

$$\vec{B} = B_{\max} \cos(kx - \omega t)\,\hat{k} \qquad Wave\ sol'ns,\ \ propagating\ in\ the\ x-dir.$$

$$\frac{\omega}{k} = \lambda f = \sqrt{\frac{1}{\mu_o \varepsilon_o}} = c \qquad the\ speed\ of\ light$$

$$E_{\max} = c\ B_{\max} \qquad coupling\ of\ the\ fields$$

Example

The earth is ~93 million miles from the sun. How long dose it take light leaving the sun to reach the earth?

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1 mile ~ 8/5 km

93 x 10^6 miles x (8/5)(1000m/mile) = 1.49 x 10^{11} m

Speed = distance/time, time = distance/speed

1.48x10^{11} m / (3x10^8 m/s) = 496 sec = 8.3 min

Example

Express the electric and magnetic fields of a sinusoidal plane electromagnetic wave having a frequency of 3.00 GHz and traveling in the positive x-direction, if the magnitude of the maximum electric field is 300 V/m.

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Express the electric and magnetic fields of a sinusoidal plane electromagnetic wave having a frequency of 3.00 GHz and traveling in the positive x-direction, if the magnitude of the maximum electric field is 300 V/m.

$$\vec{E} = E_{\max} \cos(kx - \omega t)\,\hat{j} \qquad E_{\max} = 300\ \mathrm{N/C}$$

$$\omega = \frac{\mathrm{f}}{2\pi} = \frac{3\times 10^9\ \frac{1}{\mathrm{sec}}}{2\pi} \cong 4.77\times 10^8\ \frac{\mathrm{rad}}{\mathrm{sec}}$$

$$\mathrm{c} = \frac{\omega}{k},\ k = \frac{\omega}{\mathrm{c}} = \frac{4.77\times 10^8\ \frac{\mathrm{rad}}{\mathrm{sec}}}{3\times 10^8\ \frac{\mathrm{m}}{\mathrm{sec}}} = 1.59/\mathrm{m}$$

$$\vec{E} = 300\ \mathrm{N/C} \cos\left((1.59/\mathrm{m})x - \left(4.77\times 10^8\ \tfrac{\mathrm{rad}}{\mathrm{sec}}\right)t\right)\hat{j}$$

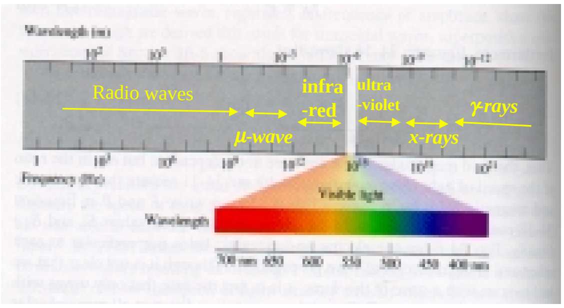
Example

Express the electric and magnetic fields of a sinusoidal plane electromagnetic wave having a frequency of 3.00 GHz and traveling in the positive x-direction, if the magnitude of the maximum electric field is 300 V/m.

$$\vec{B} = B_{\max} \cos(kx - \omega t)\,\hat{k} \qquad B_{\max} = \frac{E_{\max}}{\mathrm{c}} = \frac{300\ \mathrm{N/C}}{3\times 10^8\ \mathrm{m/s}} = 10^{-6}\ \mathrm{T}$$

$$\vec{B} = 10^{-6}\ \mathrm{T} \cos\left((1.59/\mathrm{m})x - \left(4.77\times 10^8\ \tfrac{\mathrm{rad}}{\mathrm{sec}}\right)t\right)\hat{k}$$

The Electromagnetic Spectrum



Maxwell's Equations - Energy in Waves

Recall, the parallel plate capacitor stored energy in the electric field,

$$U = \frac{1}{2}CV^2$$

And the energy density, $u_E = U/\text{Vol}$.

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

The energy is Stored in the E field.
Similarly, E&M waves carry energy.

Maxwell's Equations - Energy in Waves

The energy density of an E&M waves do the the E field is

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

From the symmetry in Maxwell's equation we can predict the energy density of the B field.

When E $\rightarrow$ B the $\varepsilon_o \rightarrow 1/\mu_o$, so

$$u_B = \frac{1}{2}\frac{B^2}{\mu_o}$$

Maxwell's Equations - Energy in Waves

The total energy density is the sum of each.

$$u_{total} = u = u_E + u_B = \frac{1}{2}\varepsilon_o E^2 + \frac{1}{2}\frac{B^2}{\mu_o}$$

Using $E = cB$ and $c^2 = 1/\varepsilon_o \mu_o$

$$u = \varepsilon_o E^2 = \frac{B^2}{\mu_o}$$

Maxwell's Equations - Energy in Waves

Because light is typically of a fast frequency, the average energy density is most commonly used.

$$u_{ave} = \varepsilon_o \langle E^2 \rangle = \frac{1}{\mu_o}\langle B^2 \rangle$$

Our fields (E and B) are sinusoidal, thus the averages are:
$\langle \cos^2(f(t)) \rangle = 1/2$

$$\boxed{u_{ave} = \frac{\varepsilon_o E_{\max}{}^2}{2} = \frac{B_{\max}{}^2}{2\mu_o}}$$

Maxwell's Equations - Energy in Waves

Thus the average energy density of a wave is given by

$$\boxed{u_{ave} = \frac{\varepsilon_o E_{\max}{}^2}{2} = \frac{B_{\max}{}^2}{2\mu_o}}$$

Units: Joules/m^3

Maxwell's Equations - Energy in Waves

The Intensity of the wave is the amount of energy per unit area per time, i.e. Watts/m^2. Since the radiation is moving at the speed of light, c, the Intensity, I, is

$$\boxed{I = cu_{ave}}$$

Example:

Solar radiation has an intensity of about 1kW/m^2 on the earth's surface. What are the peak electric and magnetic fields in solar radiation?

Example:

Solar radiation has an intensity of about 1kW/m^2 on the earth's surface. What are the peak electric and magnetic fields in solar radiation?

$$I = cu_{ave} = c\frac{\varepsilon_o E_{\max}^2}{2} \rightarrow E_{\max} = \sqrt{\frac{2I}{\varepsilon_o c}} \cong 868\,{}^{N}\!/\!{}_{C}$$

$$E_{\max} = cB_{\max} \rightarrow B_{\max} = \frac{E_{\max}}{c} \cong \frac{868\,{}^{N}\!/\!{}_{C}}{3\times10^8\,{}^{m}\!/\!{}_{s}} \cong 2.89\times10^{-6}\,\mathrm{T}$$

Example:

How much electromagnetic energy is contained per cubic meter near the Earth's surface if the intensity of Sun light under clear skies is about 1000 W/m^2?

$$U = u \times \mathrm{Vol.}$$

$$I = cu_{ave}$$

$$\rightarrow u_{ave} = \frac{I}{c} \cong \frac{1000\,{}^{W}\!/\!{}_{m^2}}{3\times10^8\,{}^{m}\!/\!{}_{s}} \cong 3.33\times10^{-6}\,{}^{J}\!/\!{}_{m^3} = 3.33\,{}^{\mu J}\!/\!{}_{m^3}$$

$$U = u \times \mathrm{Vol.} \cong 3.33\,{}^{\mu J}\!/\!{}_{m^3} \times 1\,\mathrm{m}^3 = 3.33\,\mu\mathrm{J}$$

Maxwell's Equations - Momentum and Radiation Pressure

The radiation pressure can be expressed as the Intensity by the velocity.

$$I = cu_{ave}, \quad P_{absorption} = \frac{I}{c} = u_{ave}$$

Where this is for an object which completely absorbs the radiation.
If the radiation is perfectly reflected, the pressure will double.

$$P_{reflecting} = 2u_{ave}$$

Maxwell's Equations - Momentum and Radiation Pressure

The momentum of radiation on a surface is the pressure times the surface area times the speed at which the radiation is hitting the surface.

$$\text{Momentum} \rightarrow p_{absorption} = \frac{u_{ave}Area}{c}$$

$$p_{reflection} = \frac{2u_{ave}Area}{c}$$

Example

Compare the attractive force of gravity due to the Sun on the Earth with the repulsive force of radiative pressure from Sun light.

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Compare the attractive force of gravity due to the Sun on the Earth with the repulsive force of radiative pressure from Sun light.

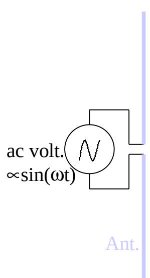
$G = 6.67 \times 10^{-11}$ N-m^2/kg^2
$R_{\odot\text{-}\oplus} = 1.5 \times 10^{11}$ m
$M_{\odot} = 2 \times 10^{30}$ kg
$M_{\oplus} = 6 \times 10^{24}$ kg
$I = 1000$ W/m^2
$R_{\oplus} = 6.4 \times 10^{6}$ m

$$P_{\text{absorption}} = \frac{I}{c}$$

$$F_{\text{Rad}} = P_{\text{absorption}}\,\text{Area} = \frac{I}{c}\pi R_{\oplus}^{2} = 4.29\times10^{8}\,\text{N}$$

$$F_{\text{Gravity}} = G\frac{M_{\oplus}M_{\odot}}{R_{\odot-\oplus}^{2}} = 3.56\times10^{22}\,\text{N}$$

Waves from Antennas



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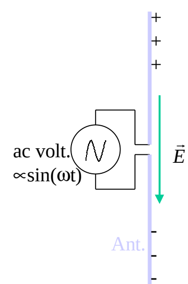

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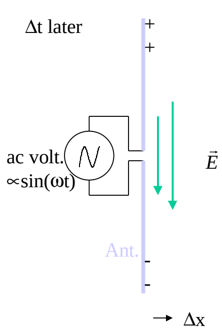

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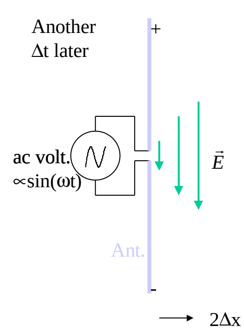

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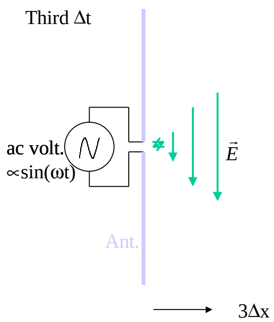
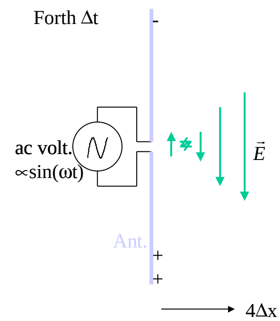


Waves from Antennas

$$v = \frac{\Delta x}{\Delta t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = c$$



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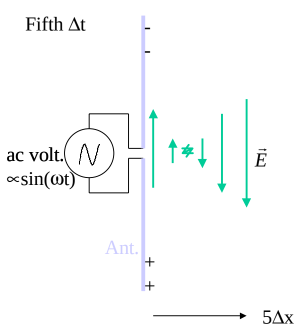

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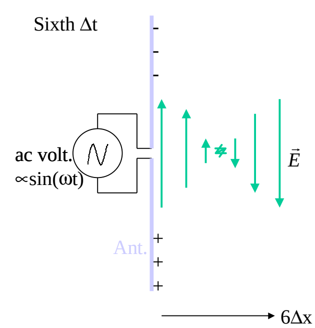

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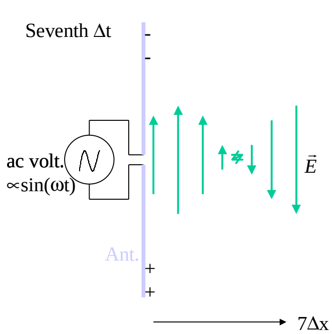

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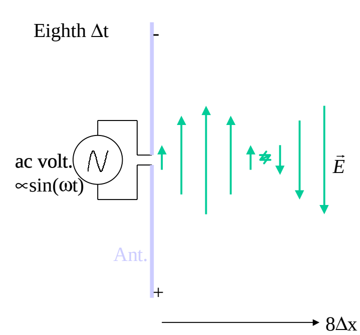

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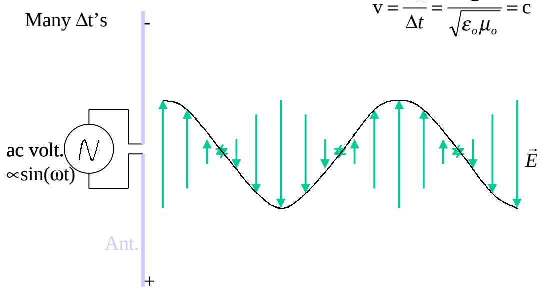

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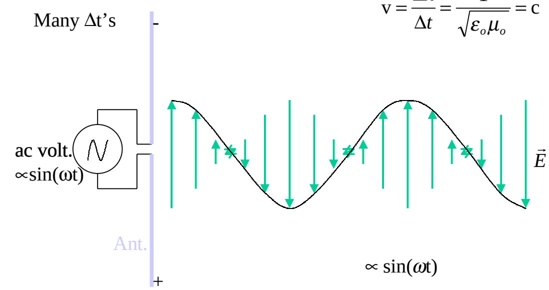
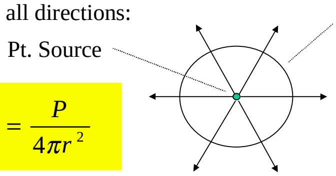
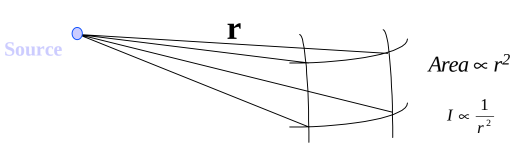


The Inverse-Square Law:

A point source of light, or any radiation, spreads out in all directions:

$$I = \frac{P}{4\pi r^2}$$





Example:
An observer is 1.8 m from a point light source whose average power P= 250 W. Calculate the rms fields in the position of the observer.

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$$I = cu_{ave} = c\frac{\varepsilon_o E_{max}^{\ 2}}{2}$$

$$I = \frac{P}{4\pi r^2}$$

$$E_{max} = \sqrt{\frac{2P}{c\varepsilon_o 4\pi r^2}} = \sqrt{\frac{P}{2c\varepsilon_o \pi r^2}}$$

$$E_{max} = \sqrt{\frac{250\,W}{2x3x10^8\frac{m}{s}x8.85x10^{-12}\frac{C^2}{N\cdot m^2}\pi(1.8m)^2}}$$

$$E_{max} \cong 68\frac{V}{m}, \quad E_{RMS} = \frac{E_{max}}{\sqrt{2}} \cong 48.1\frac{V}{m}, \quad B_{RMS} = \frac{E_{RMS}}{c} = 1.6x10^{-7}T$$