Sources of magnetic fields

Chapter 30

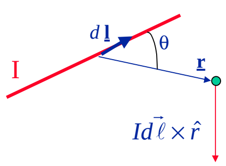
Biot-Savart Law
Lines of magnetic field
Ampere's Law
Solenoids and Toroids
Magnetic flux and Gauss's Law for Magnetism

Sources of Magnetic Fields

- We saw that magnetic fields from permanent magnets exert forces on moving charges.
- It turns out that something reciprocal happens: moving charges give rise to magnetic fields (which can then exert a force on other moving charges).
- We'll start with currents in wires, the easiest case; but it turns out that the magnetism of permanent magnets also comes from moving charges (the electrons in the atoms).

Biot-Savart Law

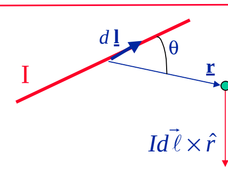
- The mathematical description of the magnetic field B due to a current is called the Biot-Savart law. It gives **B** at some position.
- A current I is moving along a path **l**. We need to add up the bits of magnetic field *d***B** arising from each infinitesimal length d**l**.



$$d\vec{B} \propto \frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Add up all the bits!

Biot-Savart Law



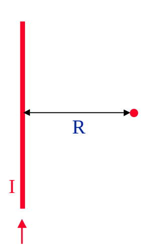
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

The constant $\mu_0 = 4\pi \times 10^{-7}$ T m/A
is called the permeability of free space.

It turns out that μ_0 and ε_o are strongly related!

Example: Magnetic field from a long wire

Consider a long straight wire carrying a current I. We want to find the magnetic field **B** at a point a distance R from the wire.



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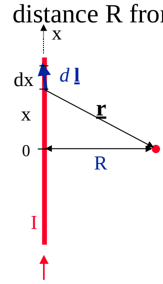
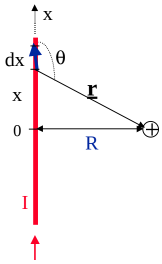


Break the wire into bits *d***l**.
To do that, choose coordinates: let the wire be along the x axis, and consider the little bit dx at a position x.

The vector **r** is from this bit to the observation point.

Example: Magnetic field from a long wire



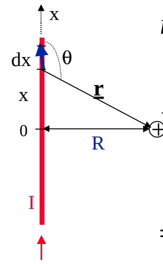
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$$

Direction: into page

$$dB = \frac{\mu_0 I}{4\pi}\frac{dx\sin\theta}{r^2}$$

$$\therefore B = \int dB = \frac{\mu_0 I}{4\pi}\int_{x=-\infty}^{x=+\infty}\frac{\sin\theta dx}{r^2}$$

Example: Magnetic field from a long wire



$$\therefore B = \int dB = \frac{\mu_0 I}{4\pi}\int_{x=-\infty}^{x=+\infty}\frac{\sin\theta dx}{r^2}$$

$$r = \sqrt{x^2 + R^2},\ \sin\theta = \frac{R}{r}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{x=-\infty}^{x=+\infty}\frac{Rdx}{(x^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi R}\left.\frac{x}{(x^2+R^2)^{1/2}}\right|_{x=-\infty}^{x=+\infty} = \frac{\mu_0 I}{2\pi R}$$

Finite Wire

See URL http://www.rwc.uc.edu/koehler/biophys/movs/bfield.mov

Magnetic field from a circular current loop

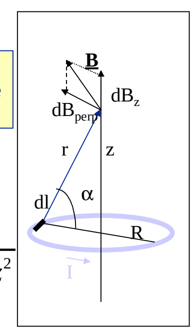
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

Only z component is nonzero over the whole loop.

$$dB_z = dB\cos\alpha = \frac{\mu_0}{4\pi}\frac{Idl\cos\alpha}{r^2}$$

$$r = \sqrt{R^2 + z^2}, \cos\alpha = R\big/\sqrt{R^2+z^2}$$



Magnetic field from a circular current loop

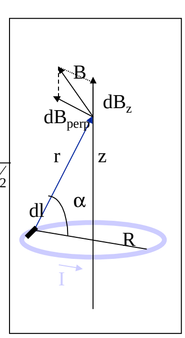
$$B = \int dB_z = \int \frac{\mu_0}{4\pi}\frac{IR}{(R^2+z^2)^{3/2}}dl$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+z^2)^{3/2}}\int_0^{2\pi R} dl =$$

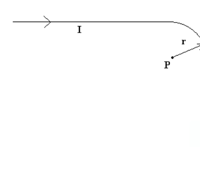
$$B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+z^2)^{3/2}}2\pi R = \frac{\mu_0}{2}\frac{IR^2}{(R^2+z^2)^{3/2}}$$

In the plane of the loop: $B = \dfrac{\mu_0 I}{2R}$

At distance z on axis from the loop, z>>R $B = \dfrac{\mu_0 IR^2}{2z^3}$

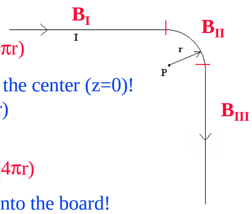


Example: Find the Magnetic Field, **B**, at the point P. The very long wire show carries a current I = 5 A and makes a 90º turn of radius r = 5 cm



Example: Find the Magnetic Field, **B**, at the point P. The very long wire show carries a current I = 5 A and makes a 90º turn of radius r = 5 cm

Break into pieces of wire and sum up the B field from each piece, i.e. superposition.



B_I is half of an infinite wire

$B_I = (1/2)\mu_o I/(2\pi r) = \mu_o I/(4\pi r)$

B_{II} is a quarter of a loop of wire at the center (z=0)!

$B_{II} = (1/4)\mu_o I/(2r) = \mu_o I/(8r)$

B_{III} is half of another infinite wire

$B_{III} = (1/2)\mu_o I/(2\pi r) = \mu_o I/(4\pi r)$

B_I, $B_{II,}$ and B_{III} are all parallel and into the board!

$B_{Total} = B_I + B_{II} + B_{III} = \mu_o I/(4\pi r) + \mu_o I/(8r) + \mu_o I/(4\pi r)$

$B_{Total} = \mu_o I/(2\pi r) + \mu_o I/(8r) = \mu_o I/((2\pi+8)r) = 8.8 \times 10^{-6}\text{T} = 8.8\ \mu\text{T}$

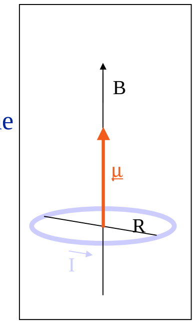
Magnetic field in terms of dipole moment

Far away on the axis,

$$B = \frac{\mu_0 IR^2}{2z^3}$$

The magnetic dipole moment of the loop is defined as $\mu = IA = I\pi R^2$.

The direction is given by another right hand rule: with fingers in the direction of the current flow, the thumb points along **μ**.



Magnetic field in terms of dipole moment

In terms of μ, the magnetic field on axis (far from the loop) is therefore

$$B = \frac{\mu_0 \mu}{2\pi z^3}$$

This also works for a loop with N turns. Far from the loop the same expression is true with the dipole moment given by $\mu = NIA = I\pi NR^2$

Dipole Equations

<u>Electric Dipole</u>

$\underline{\tau} = \underline{p} \times \underline{E}$

$U = -\underline{p} \cdot \underline{E}$

$E_{ax} = (2\pi\varepsilon_0)^{-1}\ p/z^3$

$E_{bis} = (4\pi\varepsilon_0)^{-1}\ p/x^3$

<u>Magnetic Dipole</u>

$\underline{\tau} = \underline{\mu} \times \underline{B}$

$U = -\underline{\mu} \cdot \underline{B}$

$B_{ax} = (\mu_0/2\pi)\ \mu/z^3$

$B_{bis} = (\mu_0/4\pi)\ \mu/x^3$

Ampere's Law

Ampere's law is to the Biot-Savart law exactly what Gauss's law is to Coulomb's law.

The fundamental law of electrostatics is Coulomb's law: given the sources, sum (or integrate) to get **E**.

From Coulomb's law one can derive Gauss's law

$$\oint \vec{E} \bullet d\vec{A} = q_{enclosed}/\varepsilon_0$$

This is always true, but usually useless. (It is useful in cases of high symmetry.)

Ampere's Law

The fundamental law of magnetostatics is the law of Biot-Savart: given the sources, integrate to get **B**.

From this one can derive Ampere's law:

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

line integral around some closed curve

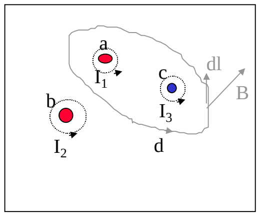
I is the sum of all the currents passing through the area enclosed by the curve. (Sign given by a right-hand rule)

This is always true, but usually useless. (It is useful in cases of high symmetry.)

Ampere's Law - a line integral

Draw an "Amperian loop" around the sources of current. The line integral of the tangential component of **B** around this loop is equal to μ_oI:

blue - into figure (-)
red - out of figure (+)



$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

HINT: TAKE ADVANTAGE OF SYMMETRY!!!!

Ampere's Law - a line integral

$$\oint_a \vec{B} \cdot d\vec{l} =$$

$$\oint_b \vec{B} \cdot d\vec{l} =$$

$$\oint_c \vec{B} \cdot d\vec{l} =$$

$$\oint_d \vec{B} \cdot d\vec{l} =$$

blue - into figure (-)
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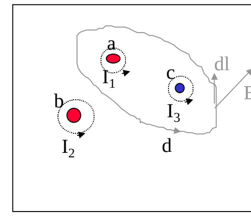


Ampere's Law - a line integral

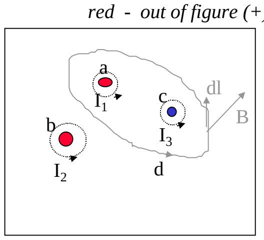
$$\oint_a \vec{B} \cdot d\vec{l} = \mu_0 I_1$$

$$\oint_b \vec{B} \cdot d\vec{l} = \mu_0 I_2$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 (-I_3)$$

$$\oint_d \vec{B} \cdot d\vec{l} = \mu_0 (I_1 - I_3)$$

blue - into figure (-)
red - out of figure (+)

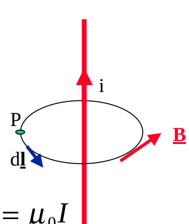


Try Ampere's Law on a wire

What is magnetic field at point P ?

Try Ampere's Law on a wire

What is magnetic field at point P ? Draw Amperian loop through P around current source and integrate **B** . dl around loop



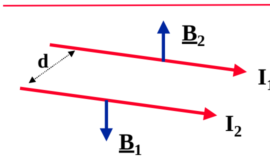
Then B · dl = Bdl
Integral is B(2πr)

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

$$B 2\pi r = \mu_0 I$$

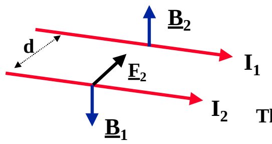
$$B = \frac{\mu_0 I}{2\pi r}$$

Force between two current-carrying wires



Current 1 produces a magnetic field $B_1 = \mu_0 I / (2\pi d)$ at the position of wire 2.

Force between two current-carrying wires



Current 1 produces a magnetic field $B_1 = \mu_0 I / (2\pi d)$ at the position of wire 2.

This produces a force on current 2:

$$\underline{F}_2 = I_2 \underline{L} \times \underline{B}_1$$

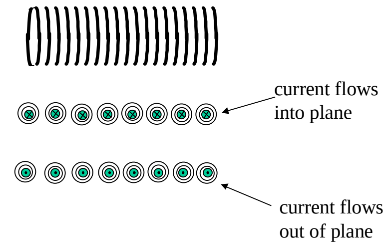
Here this gives the force on a length L of wire 2 to be:

$$F_2 = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Direction: towards 1, if the currents are in the same direction.
What is the force on wire 1?
What happens if one current is reversed?

A Solenoid

.. is a closely wound coil having n turns per unit length.



What direction is the magnetic field?

A Solenoid

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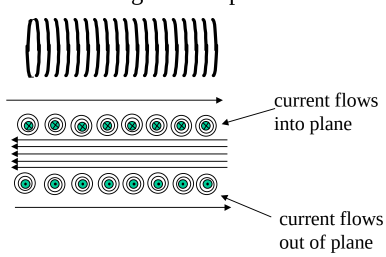
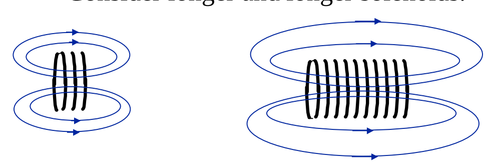
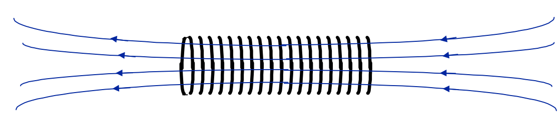


What direction is the magnetic field?

A Solenoid

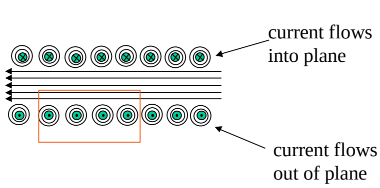
Consider longer and longer solenoids.





Fields get weaker and weaker outside.

Apply Ampere's Law to the loop shown.
Is there a net enclosed current?
In what direction does the field point?
What is the magnetic field inside the solenoid?



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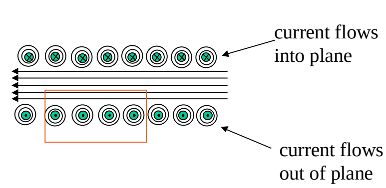


$$B(L) = \mu_0 (NI) \Rightarrow B = n\mu_0 I$$

Solenoids and Toroids

- Solenoids $B = \mu_o In$ n = # of turns/m of length of the solenoid

  This is valid inside, not too near the ends.
- A toroid is a solenoid bent in a circle. A similar calculation gives $B = \mu_o IN/2\pi r$, where in this case N is the total number of turns.

Gauss's Law for Magnetism

For electric charges Gauss's Law is $\oint \vec{E} \bullet d\vec{A} = q_{enclosed}/\varepsilon_0$

- because there are single electric charges. On the other hand, we have never detected a single magnetic charge, only dipoles. Since there are no magnetic monopoles there is no place for magnetic field lines to begin or end. Thus, Gauss's Law for magnetic charges must be

$$\oint \vec{B} \bullet d\vec{A} = 0$$

Present Status

We have now 2 1/2 of Maxwell's 4 fundamental laws of electromagnetism. They are:

Gauss's law for electric charges

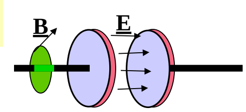
Gauss's law for magnetic charges

Ampere's law (it is still incomplete as it only applies to steady currents in its present form. Therefore, "half" of a law.)

....lets take a look at charge flowing into a capacitor....

…in Ampere's Law
...we assumed constant current...

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$



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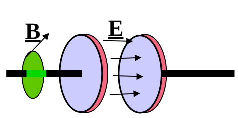
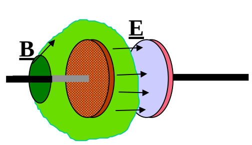
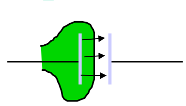


.. if the loop encloses one plate of the capacitor..



Side view:

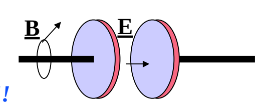
(Surface is now like a bag:)



...Maxwell solved this problem by realizing....

..inside the capacitor there is an induced magnetic field...

inside capacitor ..changing E!



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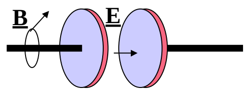

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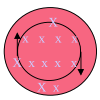


A changing electric field induces a magnetic field

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_o \frac{d\Phi_E}{dt} = \mu_0 I_d$$

where I_d is called the ...displacement current

Therefore Maxwell's revision of Ampere's Law is now.....

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I + \mu_0 \varepsilon_o \frac{d\Phi_E}{dt}$$

Derivation of Displacement Current

For a capacitor, $q = \varepsilon_0 EA$ and $I = \frac{dq}{dt} = \varepsilon_0 \frac{d(EA)}{dt}$.

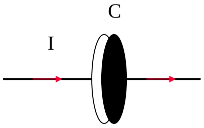
Now, the electric flux is given by EA, so: $I = \varepsilon_0 \frac{d(\Phi_E)}{dt}$, where this current , not being associated with charges, is called the "Displacement current", I_d.

Hence: $$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

and: $$\oint \vec{B} \bullet d\vec{s} = \mu_0 (I + I_d)$$

$$\Rightarrow \oint \vec{B} \bullet d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

A P.P. CAP is constructed with circular plate of cross sectional area 10.0 cm^2 and separated by 1.0 mm. The cap is in a series circuit which has 0.1 A of current flowing in it. Find the displacement current in the CAP.

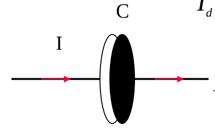


A P.P. CAP is constructed with circular plate of cross sectional area 10.0 cm^2 and separated by 1.0 mm. The cap is in a series circuit which has 0.1 A of current flowing in it. Find the displacement current in the CAP.

$$I_d = \varepsilon_0 \frac{d(\Phi_E)}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{d(E)}{dt} \qquad E = ?$$

$$q = CV = CEd \qquad E = \frac{q}{Cd} \qquad \frac{d(E)}{dt} = \frac{d}{dt}\left(\frac{q}{Cd}\right) = \frac{1}{Cd}\frac{dq}{dt} = \frac{I}{Cd}$$

$$I_d = \varepsilon_0 A \frac{I}{Cd} \qquad \text{for a PP cap. } C = \varepsilon_0 \frac{A}{d}$$



$$I_d = \varepsilon_0 A \frac{I}{d}\frac{1}{C} = \varepsilon_0 A \frac{I}{d}\frac{d}{\varepsilon_0 A} = I = 0.1A$$