

# Magnetic Fields

## Chapter 29

Permanent magnets & magnetic field lines

The magnetic force on charges

Motion of a charged particle in a magnetic field

Magnetic force on a current

Current loops in magnetic fields

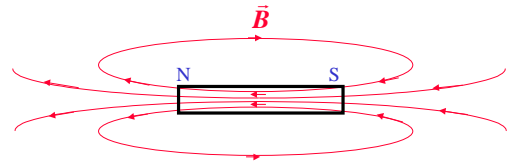
## Permanent Magnets

- Our most familiar experience of magnetic fields is through permanent magnets.
- Some materials exhibit a property we call “Ferromagnetism” - i.e. they can have a magnetic field induced in them that is permanent.
- The forces involved are large.

## What do we know about permanent magnets?

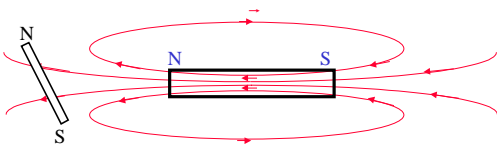
- They *always* have two poles.
- Like poles repel, opposite poles attract.
  - i.e. there are magnetic forces and *fields!*
- They also attract un-magnetised ferromagnetic materials.
- We can map out the field of a magnet using either a small magnet or small magnetic materials....

## Field of a Permanent Magnet



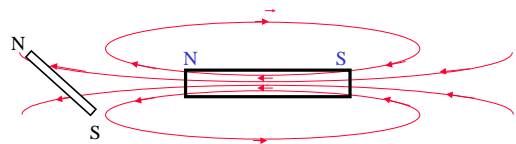
Units of magnetic field:

## Field of a Permanent Magnet



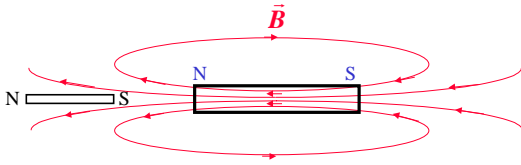
Bar magnet (magnetic dipole) wants to align to **B**-field.

## Field of a Permanent Magnet



Bar magnet (magnetic dipole) wants to align to **B**-field.

## Field of a Permanent Magnet



Bar magnet (magnetic dipole) wants to align to  $\vec{B}$ -field.

-The field exerts a torque on the dipole

## Force on a charge in a magnetic field

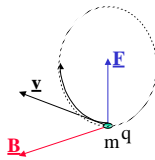
- As we saw, force is perpendicular to both  $\vec{v}$  and  $\vec{B}$ .
- The force is also largest for  $\vec{v}$  perpendicular to  $\vec{B}$ , smallest for  $\vec{v}$  parallel to  $\vec{B}$ .

This can be summarized as:

$$\vec{F} = q\vec{v} \times \vec{B}$$

or:

$$F = qvB \sin\theta$$



## The magnetic force is *different* from the electric force.

Whereas the electric force acts in the same direction as the field:

$$\vec{F} = q\vec{E}$$

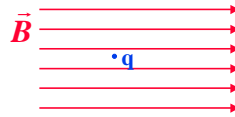
The magnetic force acts in a direction orthogonal to the field:

$$\vec{F} = q\vec{v} \times \vec{B}$$

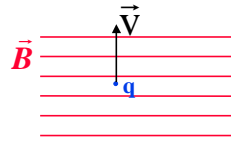
(Use "Right-Hand" Rule to determine direction of  $\vec{F}$ )

**And --- the charge must be moving !!**

## What force does a magnetic field exert on charges?



- NONE!
- (If the charge is not moving with respect to field)



- If the charge is moving, however, there is a force on the charge, *perpendicular* to both  $\vec{v}$  and  $\vec{B}$ .

## Units of magnetic field

As  $F = qvB \sin\theta$ , then,

$$B = \frac{F}{qv \sin\theta}$$

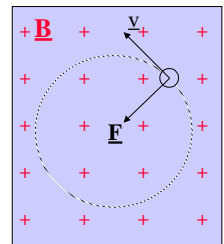
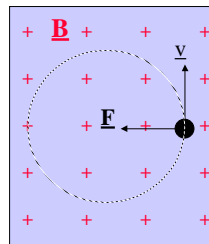
Therefore the units of magnetic field are:  $\frac{N}{C \cdot m/s} = Ns / Cm$

...OR: **Tesla, (T)    1T = 1Ns / Cm**

(Note: 1 Tesla = 10,000 Gauss)

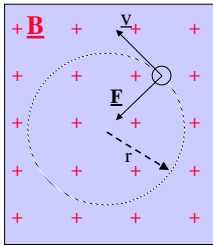
## Trajectory of charged particles in a magnetic field

( $\vec{B}$  field points into plane of paper.)



Magnetic Force is a *centripetal* force

## Radius of charged particle orbit in a magnetic field



Centripetal Force = Magnetic Force

$$\therefore \frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{mv}{qB}$$

**Note:** as  $\vec{F} \perp \vec{v}$ , the magnetic force does no work!

## Example: Mass spectrometer

Single ionized particles strike the Mass spectrometer at  $y = 10$  cm. What are the particles?

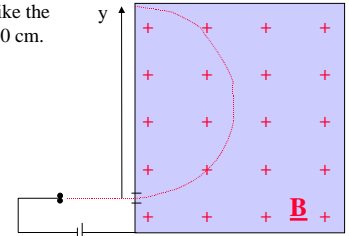
$B = 1\text{T}$   
 $V = 30\text{ kVolts}$

$$r = \frac{mv}{qB} = \frac{y}{2}$$

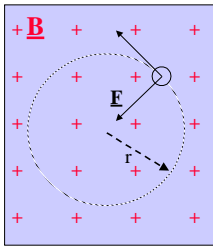
$$\frac{1}{2}mv^2 = qV$$

$$\rightarrow v = \sqrt{\frac{2qV}{m}} \quad \frac{y}{2} = \sqrt{\frac{2mV}{qB^2}} \quad m = \left(\frac{By}{2}\right)^2 \frac{q}{2V} \approx 6.65 \times 10^{-27} \text{ kg}$$

**Helium!**



## Cyclotron Frequency



The time taken to complete one orbit is:

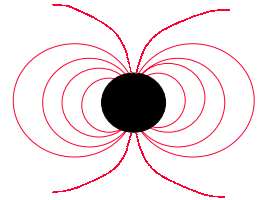
$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{v qB}$$

Hence the orbit frequency,  $f$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- known as the "cyclotron frequency"  
- can measure ratio  $e/m$

Astronomers have observed X-rays (freq.  $\sim 2 \times 10^{20}$  Hz) from the Crab Nebula at the central progenitor, a neutron star. If these X-rays are due to cyclotron radiation from electrons at the magnetic poles, what is the magnitude of the magnetic field at the poles?



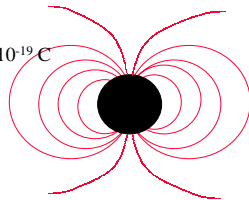
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$$f = qB/2\pi m,$$

$$B = 2\pi f m/q$$

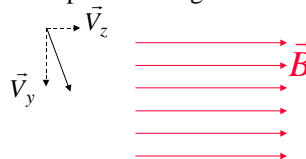
$$B \approx 2\pi \times 2 \times 10^{20} \text{ Hz} \times 9.1 \times 10^{-31} \text{ kg} / 1.6 \times 10^{-19} \text{ C}$$

$$B \approx 7.1 \times 10^9 \text{ Tesla}$$



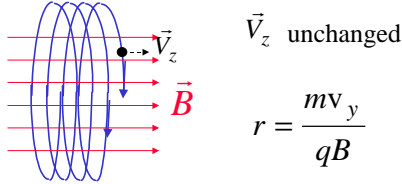
## Trajectory of charged particles in a magnetic field

What if the charged particle has a velocity component along  $B$ ?



## Trajectory of charged particles in a magnetic field

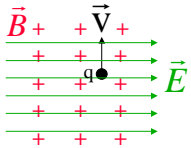
What if the charged particle has a velocity component along B?



## The electromagnetic force:

If a magnetic field and an electric field are simultaneously present, their *forces* obey the superposition principle and may be added as vectors:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

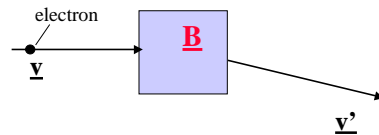


—Here,  $\vec{F}_E$  and  $\vec{F}_B$  point in opposite directions

## The electromagnetic force:

If a magnetic field and an electric field are simultaneously present, their *forces* obey the superposition principle and may be added as vectors:

## Exercise:



- In what direction does the magnetic field point?
- Which is bigger,  $v^2$  or  $v'^2$  ?

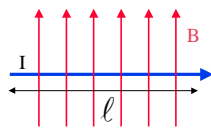
## Magnetic force on a current

- We find that a magnetic field exerts a force on a current. ---WHY?
  - Remember, a current is a *flow of charge*, so charge is moving at the drift velocity,  $v_d$ .
  - $v_d$  is small, but there are many charges.

We have,

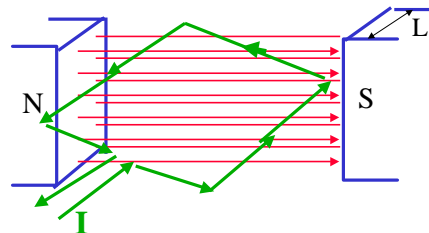
$$\vec{F} = q\vec{v} \times \vec{B} = q \frac{\Delta \vec{x}}{\Delta t} \times \vec{B}$$

$$= \frac{\Delta q}{\Delta t} \Delta \vec{x} \times \vec{B} = i \vec{l} \times \vec{B}$$

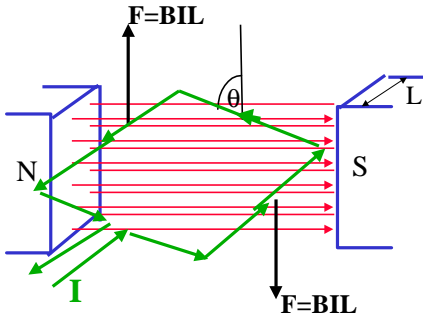


## Magnetic force on a current -Torque & electric motor

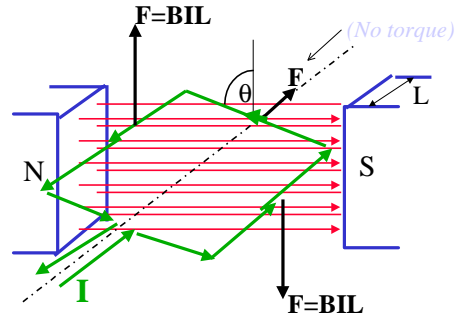
Consider the current through a loop in a uniform magnetic field:



## Magnetic force on a current -Torque & electric motor

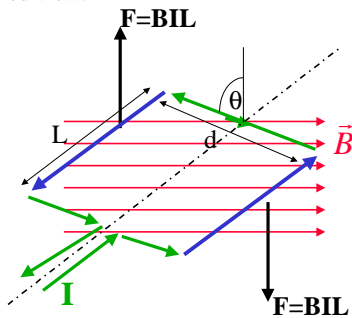


## Magnetic force on a current -Torque & electric motor



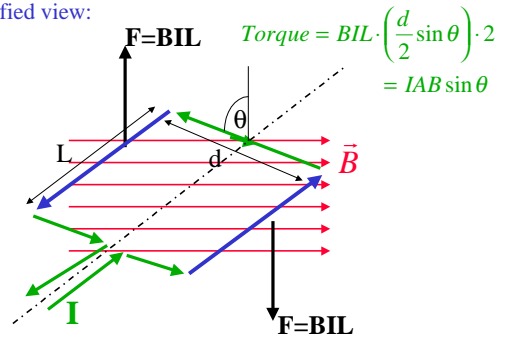
## Magnetic force on a current - Torque & electric motor

Simplified view:

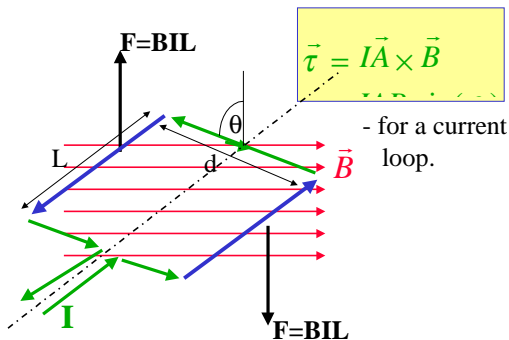


## Magnetic force on a current - Torque & electric motor

Simplified view:

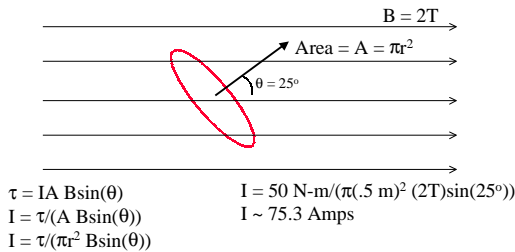


## Magnetic force on a current - Torque & electric motor



Example: a circular coil ( $r = 0.5$  m) of current  $I$ , experience a torque of  $50$  N-m due to a constant magnetic field,  $B = 2$  T. If the magnetic field is making an angle of  $25^\circ$  with respect to the area vector of the coil, what is the current in the coil?

Example: a circular coil ( $r = 0.5 \text{ m}$ ) of current  $I$ , experience a torque of  $50 \text{ N-m}$  due to a constant magnetic field,  $B = 2\text{T}$ . If the magnetic field is making an angle of  $25^\circ$  with respect to the area vector of the coil, what is the current in the coil?



## Magnetic force on a current - Torque & Magnetic Dipole

By analogy with electric dipoles, for which:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The expression,

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

implies that a current loop acts as a **magnetic dipole!**

Where,  $\vec{\mu} = IA$ , is the magnetic dipole moment.

Hence,

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (\text{Torque on a current loop})$$

## Multiple-loop Magnetic Dipoles

A current loop acts as a **magnetic dipole!**

Where  $\vec{\mu} = IA$ , is the magnetic dipole moment.

If a coil is formed of many ( $N$ ) loops, the magnetic dipole moment is:

$$\vec{\mu} = NI\vec{A}$$

## Energy of Magnetic Dipole

By further analogy with electric dipoles:

$$U_E = -\vec{p} \cdot \vec{E}$$

So for a magnetic dipole, (current loop)

$$U_B = -\vec{\mu} \cdot \vec{B}$$

Where  $\vec{\mu} = NI\vec{A}$ , is the magnetic dipole moment.

Example: a coil of current  $I$ , experience a torque of  $50 \text{ N-m}$  due to a constant magnetic field,  $B = 2\text{T}$ . If the magnetic field is making an angle of  $25^\circ$  with respect to the area vector of the coil, what is the magnetic dipole moment of the coil?

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$$\vec{\mu} = NI\vec{A}$$

