Electric Fields
i.e. The Electric Charge, Electric Force, and Electrostatics

Chapter 23

Electric Charge

History

600 BC	Greeks first discover attractive properties of amber when rubbed.
1600 AD	Electric bodies repel as well as attract.

Electric Charge

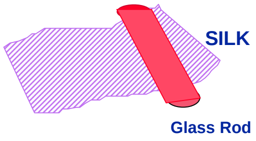
The Charge

Two types of charge: **+ protons**
- electrons

Like charges repel each other.
Opposites attract each other.

Electric Charge

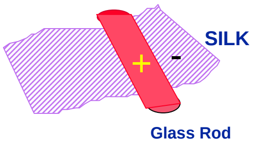
The Transfer of Charge



Some materials attract electrons more than others.

Electric Charge

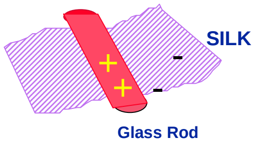
The Transfer of Charge



Here electrons are pulled off the glass onto the silk.

Electric Charge

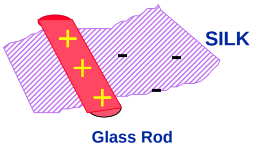
The Transfer of Charge



Usually matter is charge neutral because the number of electrons and protons are equal. But here the silk has an excess of electrons and the rod a deficit.

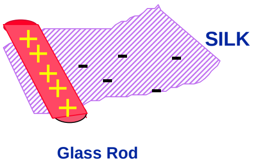
Electric Charge

The Transfer of Charge



Electric Charge

The Transfer of Charge



Glass and silk are insulators:
charges stuck on them stay put.

Electric Charge

Conductors Vs. Insulators

In *conductors,* charges are free to move about. The charges (usually electrons) arrange themselves into a static situation ($\mathbf{F}_{tot} = 0$). Conductors with a net charge (positive or negative) have the excess charge move to the surface, if no other forces are present. Metals are conductors!

In *insulators,* charges cannot move freely. The charges stay where they are put.

Electric Charge

History

600 BC	Greeks first discover attractive properties of amber when rubbed.
1600 AD	Electric bodies repel as well as attract
1735 AD	du Fay: Two distinct types of electricity
1750 AD	Franklin: Positive and Negative Charge
1770 AD	Coulomb: “Inverse Square Law”
1890 AD	J.J. Thompson: Quantization of electric charge - “Electron”

Electric Charge

Summary of things we know:

- There is a property of matter called electric charge. (In the SI system its units are Coulombs.)
- Charges can be negative (like electrons) or positive (like protons).
- In matter, the positive charges are stuck in place in the nuclei. Matter is negatively charged when extra electrons are added, and positively charged when electrons are removed.
- Like charges repel, unlike charges attract.
- Charges travel in conductors, not in insulators
- Force of attraction or repulsion ~ $1 / r^2$

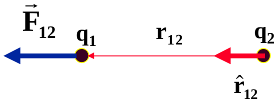
and ... Charge is Quantized

q = multiple of an elementary charge e:
$e = 1.6 \times 10^{-19}$ Coulombs

	Charge	Mass	Diameter
electron	- e	1	0
proton	+e	1836	$\sim 10^{-15}$m
neutron	0	1839	$\sim 10^{-15}$m
positron	+e	1	0

(Protons and neutrons are made up of quarks, whose charge is quantized in multiples of ± ne/3. Quarks can’t be isolated under normal conditions.)

Coulomb's Law



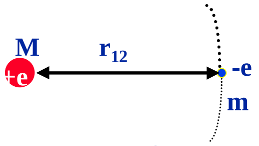
$$\vec{\mathbf{F}}_{12} = \frac{k q_1 q_2}{r_{12}^{2}}\,\hat{\mathbf{r}}_{12}$$

Force on 1 due to 2

$k = (4\pi\varepsilon_0)^{-1} = 9.0 \times 10^{9}\ \mathrm{Nm^2/C^2}$

ε_0 = permitivity of free space = $8.86 \times 10^{-12}\ \mathrm{C^2/Nm^2}$

Gravitational and Electric Forces of the Hydrogen atom

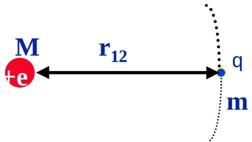


$m = 9.1\ 10^{-31}$ kg
$M = 1.7\ 10^{-27}$ kg
$r_{12} = 5.3\ 10^{-11}$ m

Gravitational force

Electric Force

Gravitational and Electric Forces of the Hydrogen atom



$m = 9.1\ 10^{-31}$ kg
$M = 1.7\ 10^{-27}$ kg
$r_{12} = 5.3\ 10^{-11}$ m
$Q = -q = 1.6\ 10^{-19}$C

Gravitational force

$$\vec{F}_g = G\frac{Mm}{r_{12}^2}\hat{r}$$

$$\boxed{F_g = 3.6\ 10^{-47}\ \mathrm{N}}$$

Electric Force

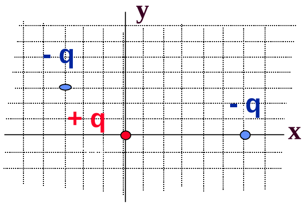
$$\vec{F}_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Qq}{r_{12}^2}\hat{r}$$

$$\boxed{F_e = 8.2\ 10^{-8}\mathrm{N}}$$

Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

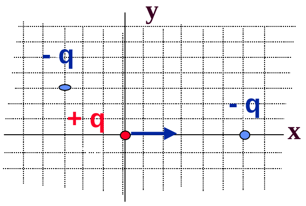


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Consider effect of each charge separately:

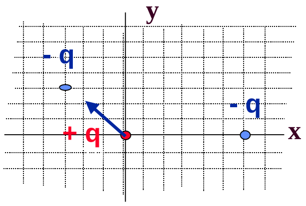


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Take each charge in turn:

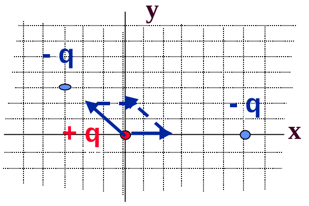


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Create vector sum:

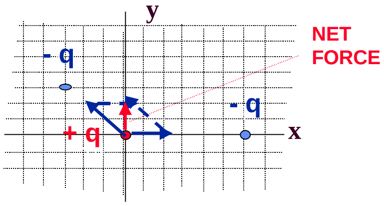


Superposition of forces from two charges

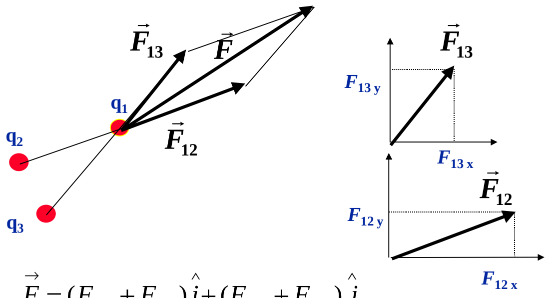
Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Find resultant:



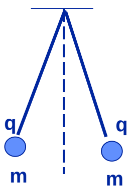
Superposition Principle



$$\vec{F} = (F_{12x} + F_{13x})\,\hat{i} + (F_{12y} + F_{13y})\,\hat{j}$$

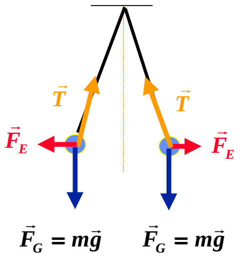
Example: electricity balancing gravity

- What forces are acting on the charged balls ?
- Write the forces in Cartesian coordinates.
- Equate forces (Newton's 3rd law)
- Solve for unknowns!



Example: electricity balancing gravity

- Draw vector force diagram while identifying the forces.
- Apply Newton's 3rd Law for a system in equilibrium to the components of the forces.
- Solve!

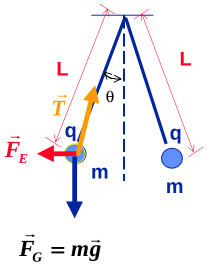


Example: electricity balancing gravity

If θ = 20°, m = 3 mg, and L = 0.75 m, what is q and the forces are acting on the charged balls?

$$\left|\vec{F}_E\right| = k\frac{q_1 q_2}{{}^2}$$

$$\left|\vec{F}_E\right| = k\frac{q^2}{{}_2\quad{}_2}$$



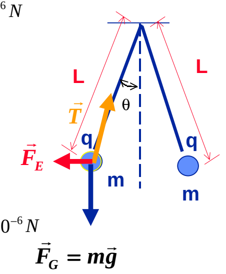
Example: electricity balancing gravity

$$\left|\vec{F}_G\right| = mg = \left(3\times10^{-6}\,kg\right)\left(9.8\,\tfrac{m}{s^2}\right) = 2.94\times10^{-6}\,N$$

$$\text{also, } \left|\vec{T}\right| = \left|\vec{F}_G\right| \Big/ \cos\theta = \frac{mg}{\cos\theta}$$

$$= \frac{2.94\times10^{-6}\,N}{\cos(20^o)} = 3.13\times10^{-6}\,N$$

$$\left|\vec{F}_E\right| = \left|\vec{T}\right|\sin\theta = 3.13\times10^{-6}\,N\sin(20^o) = 1.07\times10^{-6}\,N$$

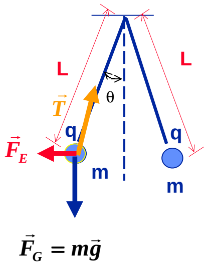


Example: electricity balancing gravity

$$\left|\vec{F}_E\right| = k\frac{q^2}{4L^2\sin^2\theta} \qquad q = \sqrt{\frac{\left|\vec{F}_E\right|4L^2\sin^2\theta}{k}}$$

$$q = \sqrt{\frac{\left(1.07\times10^{-6}\,N\right)4\left(0.75m\right)^2\sin^2(20^o)}{9\times10^9\ ^{Nm^2}\!/_{C^2}}}$$

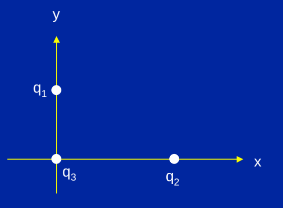
$$q \cong 7.19\times10^{-9}\,C$$



example problem

Example: Three point charges are arranged as shown in the figure. Find the total force on 3 due to 1 and 2.

Assume: q_1 = -1 x 10^{-8} C, position (0, 0.5 m)
q_2 = 5 x 10^{-8} C, position (0.8 m, 0)
q_3 = 2 x 10^{-8} C, position (0, 0)



example problem

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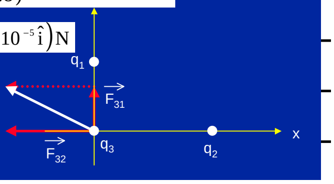
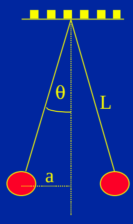
$$\vec{\mathrm{F}}_{31} = \frac{kq_3q_1}{r_{31}^2}\hat{\mathbf{r}}_{31} = \frac{\left(9\times10^9\right)\left(2\times10^{-8}\right)\left(-1\times10^{-8}\right)}{(0.5)^2}\mathbf{N}\left(-\hat{\mathbf{j}}\right) \cong 7.2\times10^{-6}\,\mathbf{N}\,\hat{\mathbf{j}}$$

$$\vec{\mathrm{F}}_{32} = \frac{kq_3q_2}{r_{32}^2}\hat{\mathbf{r}}_{32} = \frac{\left(9\times10^9\right)\left(2\times10^{-8}\right)\left(5\times10^{-8}\right)}{(0.8)^2}\mathbf{N}\left(-\hat{\mathbf{i}}\right) \cong -1.4\times10^{-5}\,\mathbf{N}\,\hat{\mathbf{i}}$$

$$\vec{\mathrm{F}} = \vec{\mathrm{F}}_{31} + \vec{\mathrm{F}}_{32} = \left(7.2\times10^{-6}\,\hat{\mathrm{j}} - 1.4\times10^{-5}\,\hat{\mathrm{i}}\right)\mathrm{N}$$
$$= \left(-1.4\,\hat{\mathrm{i}} + 0.72\,\hat{\mathrm{j}}\right)\times10^{-5}\,\mathrm{N}$$



Example: Two Charged Spheres



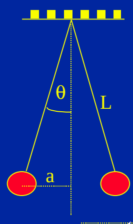
What is the charge on each sphere?

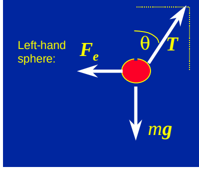
Mass of each sphere is 30 g. L=15 cm. θ = 5^0.

From geometry we see that
a = L sin θ = (0.15 m) sin 5^0 = 0.013 m

Therefore the separation of the spheres is 2a = 0.026 m

Example: Two Charged Spheres





What is the charge on each sphere?

Mass of each sphere is 30 g. L=15 cm. θ = 5^0.

The total force on each sphere is zero:

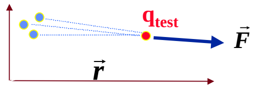
F_y = 0 = T cos θ - mg so T =mg/cosθ
F_x = 0 = T sin θ - F_e
so F_e= Tsinθ = mg tanθ

But also $F_e = kq^2/r^2$ (**Coulomb's law**).
Equating $F_e = F_e$ gives
kq^2/r^2 = mg tanθ gives
$q^2 = (1/k)\,r^2$ mg tan θ

$\boxed{q = 4.4\ 10^{-8}\ \mathrm{C}}$

The Electric Field

- A set of fixed charges exerts a force $\vec{F}$ given by Coulomb's law on a test charge q_{test} at position $\vec{r}$



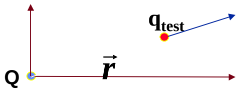
- The electric field is represented by the symbol $\vec{E}$, and is given in terms of this force by:

$$\vec{E}(\vec{r}) = \frac{\vec{F}}{q_{test}}$$

This is a vector function of position.

Example: Point Charge

- Find the electric field of a point charge Q located at the origin.



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_{test}}{r^2}\hat{r}$$

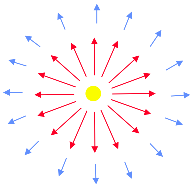
- Dividing out q_{test} gives the electric field at $\vec{r}$:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

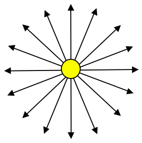
Radially outward, falling off as $1/r^2$

Example: Point Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



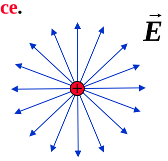
Vector Field
(Show $\vec{E}$ at each $\vec{r}$)



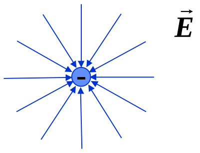
Field Lines
(Lines of force)

The Electric Field

- The concept of electric fields was invented by Michael Faraday to describe his model of how charges interact.
- Charges interact by exerting forces on each other. Faraday thought of the fields as "lines of force" (sort of like strings) with a density in space proportional to the strength of the force.



Positive charge ***(Source)***

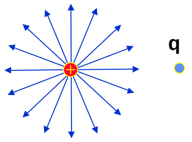


Negative charge ***(Sink)***

Force due to an electric field

Just turn the definition of $\vec{E}$ around.
If $\vec{E}(\vec{r})$ is known, the force on a charge q at point $\vec{r}$ is:

$$\vec{F} = q\vec{E}(\vec{r})$$



The electric field at $\vec{r}$ points in the direction that a positive charge placed at $\vec{r}$ would be pushed.

Electric field lines are bunched closer where the field is stronger!

Electric fields due to various charge distributions

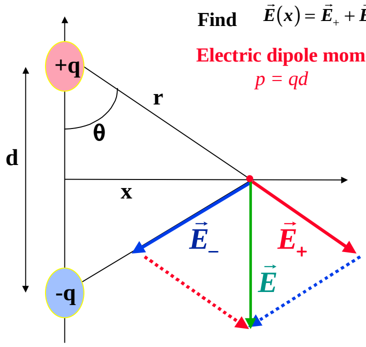
The electric field is a vector which obeys the superposition principle.

Begin with a simple example with discrete charges: the dipole

Field Due to an Electric Dipole

at a point x straight out from its midpoint

Find $\vec{E}(x) = \vec{E}_+ + \vec{E}_-$



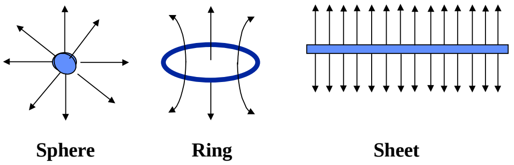
Electric dipole moment

p = qd

Electric fields from continuous distributions

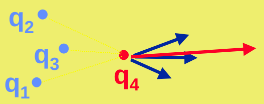
Up to now we have only considered the electric field of point charges.

Now let’s look at continuous distributions of charge --- lines or surfaces or volumes of charge --- and determine the resulting electric fields.



Electric fields from continuous distributions

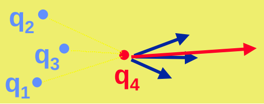
For discrete point charges, we can use the superposition principle and sum the fields due to each point charge:



$$\vec{E}(\vec{r}) = \sum_i \vec{E}_i$$

Electric fields from continuous distributions

For discrete point charges, we can use the superposition principle and sum the fields due to each point charge:

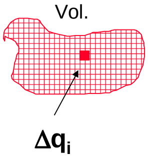


$$\vec{E}(\vec{r}) = \sum_i \vec{E}_i$$

What if we now have a *continuous* charge distribution?

Electric fields from continuous distributions

- We divide the distribution up into small pieces, then we sum the contribution to the field from each piece:

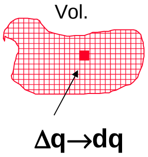


$$\Delta \vec{E}_i(\vec{r}) = k\frac{\Delta q_i}{r^2}\hat{r}$$

$$\vec{E}(\vec{r}) \approx \sum_i \Delta \vec{E}_i(\vec{r}) = \sum_i k\frac{\Delta q_i}{r^2}\hat{r}$$

In the limit of very small pieces, the sum is an *integral*

Electric fields from continuous distributions



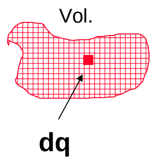
$$\vec{E}(\vec{r}) = \lim_{\Delta q_i \to 0} \sum_{i,vol} k\frac{\Delta q_i}{r^2}\hat{r}$$

$$\vec{E}(\vec{r}) = \int_{Vol.} k\frac{dq}{r^2}\hat{r}$$

Hence, $\vec{E}(\vec{r}) = \int d\vec{E},\ d\vec{E} = k\frac{dq}{r^2}\hat{r}$

Electric fields from continuous distributions

Hence, $\vec{E}(\vec{r}) = \int d\vec{E}$

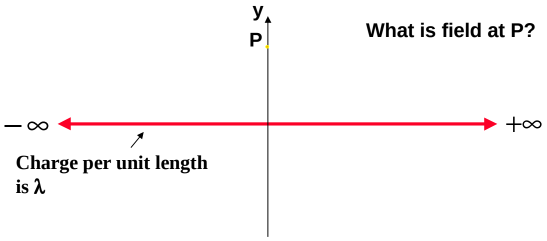


$$d\vec{E}(\vec{r}) = k\frac{dq}{r^2}\hat{r}$$

$$\vec{E}(\vec{r}) = \int_{Vol.} k\frac{dq}{r^2}\hat{r}$$

Example: An infinite thin line of charge.

What is field at P?



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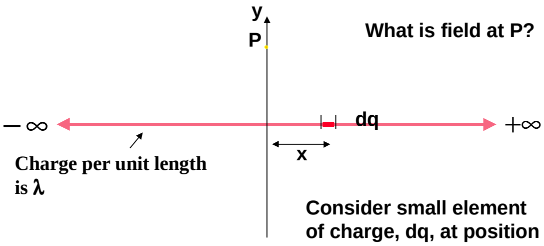
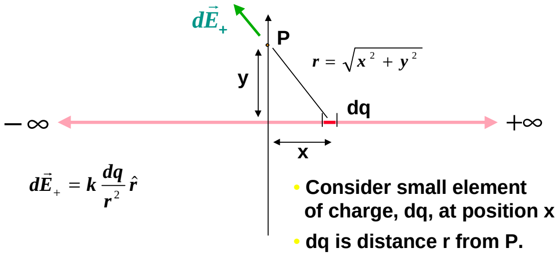


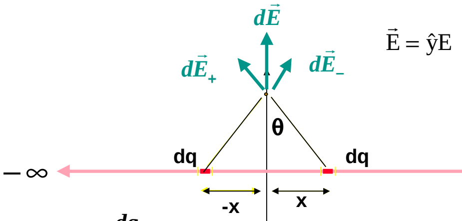
Consider small element of charge, dq, at position x.

Example: An infinite thin line of charge.



$$d\vec{E}_+ = k\frac{dq}{r^2}\hat{r}$$

- Consider small element of charge, dq, at position x.
- dq is distance r from P.
- dq produces dE at P.



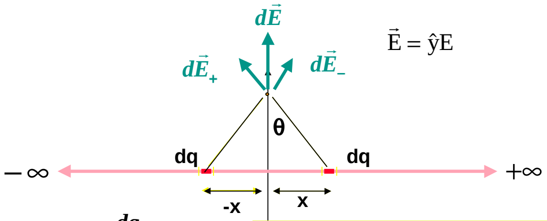
$\vec{E} = \hat{y}E$

$$d\vec{E}_+ = k\frac{dq}{r^2}\hat{r}$$

$$dE_y = 2k\frac{dq}{r^2}\cos\theta$$

dq=λ dx, cosθ=y/r

- Consider small element of charge, dq, at position x.
- dq is distance r from P.
- dq produces dE at P.
- For each dq at +x, there is a dq at -x.



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$$dE_y = 2k\frac{dq}{r^2}\cos\theta$$

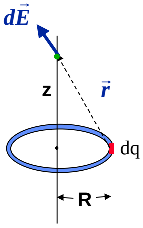
dq=λ dx, cosθ=y/r

$$dE_y = \frac{2k\lambda dx}{(x^2+y^2)}\cdot\frac{y}{r}$$

$$E_y = \int_{x=0}^{x=\infty}\frac{2k\lambda y}{(x^2+y^2)^{3/2}}dx = \frac{2k\lambda}{y}$$

Example of continuous distribution: ring of charge

Find the electric field at a point along the axis.
Hint: be sure to use the symmetry of the problem!



Thin ring with total charge q: charge per length is

$$\lambda = \frac{q}{2\pi R}$$

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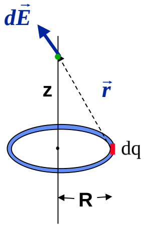


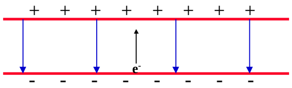
Thin ring with total charge q: charge per length is

$$\lambda = \frac{q}{2\pi R}$$

Break the charge up into little bits and find the field due to each bit at the observation point. Then integrate. This is an important example problem in the Text!

Chap 23: Example

A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time 1.5 x 10^{-6} sec. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?



$$x = x_0 + v_0 t + \frac{1}{2}\mathrm{a}t^2$$

$$2cm = 0 + 0 + \frac{1}{2}\mathrm{a}\left(1.5\times10^{-6}\,s\right)^2$$

$$\mathrm{a} = 1.78\times10^{12}\ \tfrac{cm}{s^2}$$

(a) $\mathrm{v}^2 - \mathrm{v}_0^2 = 2\mathrm{a}(\mathrm{x}-\mathrm{x}_0)$

$\mathrm{v}^2 = 2\mathrm{a}(\mathrm{x}) \rightarrow \mathrm{v} = \sqrt{2\mathrm{a}(\mathrm{x})}$

$\mathrm{v} = 2.6\times10^{6}\ \tfrac{cm}{s}$

(b) $\mathrm{F} = \mathrm{ma} = \mathrm{qE},\ \mathrm{E} = \dfrac{\mathrm{ma}}{\mathrm{q}}$

$$\mathrm{E} = \frac{9.11\times10^{-31}\,\mathrm{kg}\left(1.78\times10^{10}\ \tfrac{m}{s^2}\right)}{1.6\times10^{-19}\,C} \cong 0.1\tfrac{\mathrm{N}}{\mathrm{C}}$$