EXPERIMENT 1

Experimental Uncertainty (Error) and Data Analysis

INTRODUCTION AND OBJECTIVES

Laboratory investigations involve taking measurements of physical quantities, and the process of taking any measurement always involves some experimental uncertainty or error.* Suppose you and another person independently took several measurements of the length of an object. It is highly unlikely that you both would come up with exactly the same results. Or you may be experimentally verifying the value of a known quantity and want to express uncertainty, perhaps on a graph. Therefore, questions such as the following arise:

- Whose data are better, or how does one express the degree of uncertainty or error in experimental measurements?
- How do you compare your experimental result with an accepted value?
- How does one graphically analyze and report experimental data?

In this introductory study experiment, types of experimental uncertainties will be examined, along with some methods of error and data analysis that may be used in subsequent experiments.

After performing the experiment and analyzing the data, you should be able to do the following:

1. Categorize the types of experimental uncertainty (error), and explain how they may be reduced.
2. Distinguish between measurement accuracy and precision, and understand how they may be improved experimentally.
3. Define the term *least count* and explain the meaning and importance of significant figures (or digits) in reporting measurement values.
4. Express experimental results and uncertainty in appropriate numerical values so that someone reading your report will have an estimate of the reliability of the data.
5. Represent measurement data in graphical form so as to illustrate experimental data and uncertainty visually.

*Although *experimental uncertainty* is more descriptive, the term *error* is commonly used synonymously.

EQUIPMENT NEEDED

- Rod or other linear object less than 1 m in length
- Four meter-long measuring sticks with calibrations of meter, decimeter, centimeter, and millimeter, respectively†
- Pencil and ruler
- Hand calculator
- 3 sheets of Cartesian graph paper
- French curve (optional)

†A 4-sided meter stick with calibrations on each side is commercially available from PASCO Scientific.

THEORY

A. Types of Experimental Uncertainty

Experimental uncertainty (error) generally can be classified as being of two types: (1) random or statistical error and (2) systematic error. These are also referred to as (1) indeterminate error and (2) determinate error, respectively. Let's take a closer look at each type of experimental uncertainty.

Random (Indeterminate) or Statistical Error

Random errors result from unknown and unpredictable variations that arise in all experimental measurement situations. The term *indeterminate* refers to the fact that there is no way to determine the magnitude or sign (+, too large; –, too small) of the error in any individual measurement. Conditions in which random errors can result include:

1. Unpredictable fluctuations in temperature or line voltage.
2. Mechanical vibrations of an experimental setup.
3. Unbiased estimates of measurement readings by the observer.

Repeated measurements with random errors give slightly different values each time. The effect of random errors may be reduced and minimized by improving and refining experimental techniques.

Systematic (Determinate) Errors

Systematic errors are associated with particular measurement instruments or techniques, such as an improperly calibrated instrument or bias on the part of the observer. The term *systematic* implies that the same magnitude and sign of experimental uncertainty are obtained when the measurement is repeated several times. *Determinate* means that the magnitude and sign of the uncertainty can be determined if the error is identified. Conditions from which systematic errors can result include

1. An improperly "zeroed" instrument, for example, an ammeter as shown in ● Fig. 1.1.
2. A faulty instrument, such as a thermometer that reads 101 °C when immersed in boiling water at standard atmospheric pressure. This thermometer is faulty because the reading should be 100 °C.
3. Personal error, such as using a wrong constant in calculation or always taking a high or low reading of a scale division. Reading a value from a measurement scale generally involves aligning a mark on the scale. The alignment—and hence the value of the reading—can depend on the position of the eye (parallax). Examples of such personal systematic error are shown in ● Fig. 1.2.

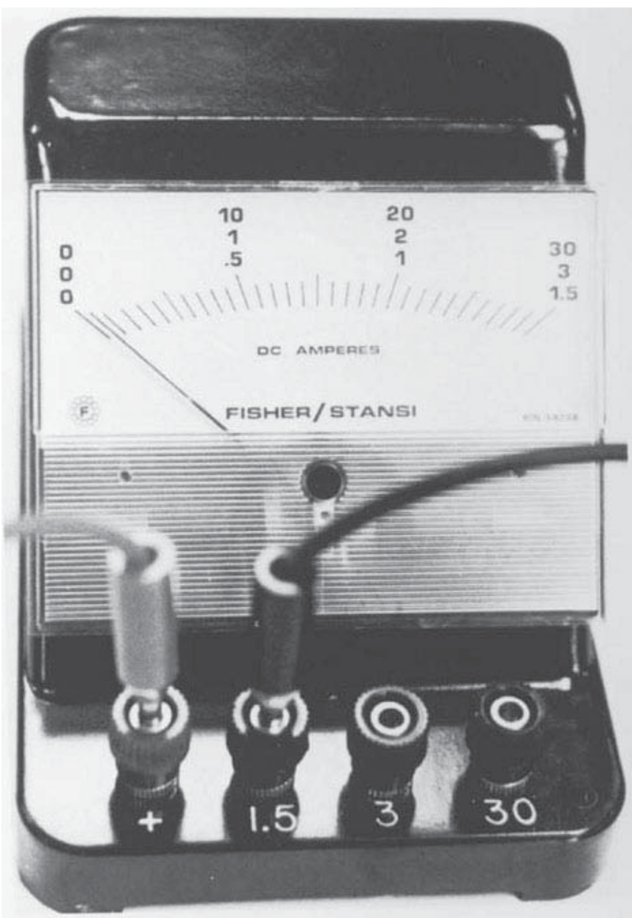


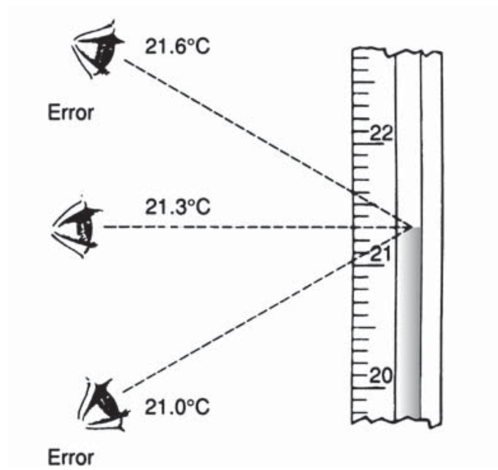
Figure 1.1 Systematic error. An improperly zeroed instrument gives rise to systematic error. In this case the ammeter, which has no current through it, would systematically give an incorrect reading larger that the true value. (After correcting the error by zeroing the meter, which scale would you read when using the ammeter?)

Avoiding systematic errors depends on the skill of the observer to recognize the sources of such errors and to prevent or correct them.

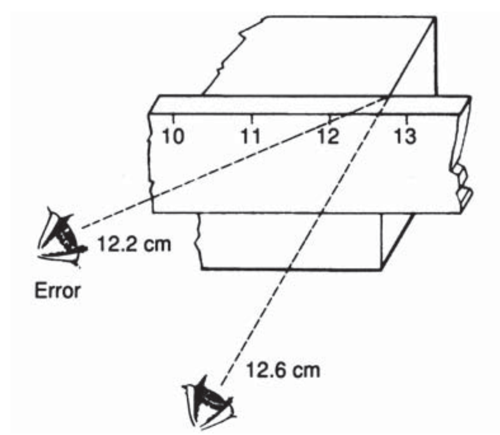
B. Accuracy and Precision

Accuracy and *precision* are commonly used synonymously, but in experimental measurements there is an important distinction. The **accuracy** of a measurement signifies how close it comes to the true (or accepted) value—that is, how nearly correct it is.

Example 1.1 Two independent measurement results using the diameter *d* and circumference *c* of a circle in the determination of the value of π are 3.140 and 3.143. (Recall that $\pi = c/d$.) The second result is



(a) Temperature measurement



(b) Length measurement

Figure 1.2 Personal error. Examples of personal error due to parallax in reading (a) a thermometer and (b) a meter stick. Readings may systematically be made either too high or too low.

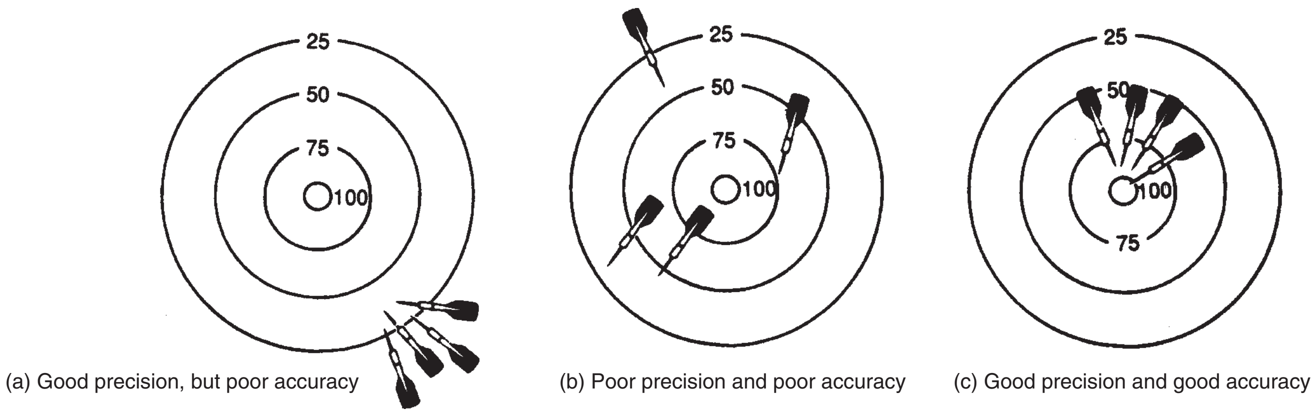


Figure 1.3 Accuracy and precision. The true value in this analogy is the bull's eye. The degree of scattering is an indication of precision—the closer together a dart grouping, the greater the precision. A group (or symmetric grouping with an average) close to the true value represents accuracy.

more accurate than the first because the true value of π, to four figures, is 3.142.

Precision refers to the agreement among repeated measurements—that is, the "spread" of the measurements or how close they are together. The more precise a group of measurements, the closer together they are. However, a large degree of precision does not necessarily imply accuracy, as illustrated in ● Fig. 1.3.

Example 1.2 Two independent experiments give two sets of data with the expressed results and uncertainties of 2.5 ± 0.1 cm and 2.5 ± 0.2 cm, respectively.

The first result is more precise than the second because the spread in the first set of measurements is between 2.4 and 2.6 cm, whereas the spread in the second set of measurements is between 2.3 and 2.7 cm. That is, the measurements of the first experiment are less uncertain than those of the second.

Obtaining *greater accuracy* for an experimental value depends in general on *minimizing systematic errors*. Obtaining *greater precision* for an experimental value depends on *minimizing random errors*.

C. Least Count and Significant Figures

In general, there are *exact* numbers and *measured* numbers (or quantities). Factors such as the 100 used in calculating percentage and the 2 in $2\pi r$ are exact numbers. Measured numbers, as the name implies, are those obtained from measurement instruments and generally involve some error or uncertainty.

In reporting experimentally measured values, it is important to read instruments correctly. The degree of uncertainty of a number read from a measurement instrument depends on the quality of the instrument and the fineness of its measuring scale. When reading the value from a calibrated scale, only a certain number of figures or digits can properly be obtained or read. That is, only a certain number of figures are *significant*. This depends on the **least count** of the instrument scale, which is the smallest subdivision on the measurement scale. This is the unit of the smallest reading that can be made without estimating. For example, the least count of a meter stick is usually the millimeter (mm). We commonly say "the meter stick is calibrated in centimeters (numbered major divisions) with a millimeter least count." (See ● Fig. 1.4.)

The **significant figures** (sometimes called **significant digits**) of a measured value include all the numbers that can be read directly from the instrument scale, *plus* one doubtful or estimated number—the fractional part of the least count smallest division. For example, the length of the rod in Fig. 1.4 (as measured from the zero end) is 2.64 cm. The rod's length is known to be between 2.6 cm and 2.7 cm. The estimated fraction is taken to be 4/10 of

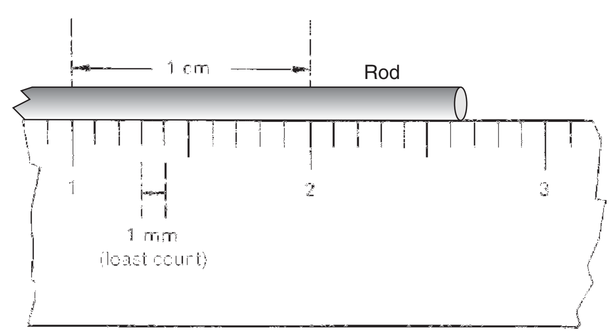


Figure 1.4 Least count. Meter sticks are commonly calibrated in centimeters (cm), the numbered major divisions, with a least count, or smallest subdivision, of millimeters (mm).

the least count (mm), so the doubtful figure is 4, giving 2.64 cm with three significant figures.

Thus, measured values contain inherent uncertainty or doubtfulness because of the estimated figure. However, the greater the number of significant figures, the greater the reliability of the measurement the number represents. For example, the length of an object may be read as 3.65 cm (three significant figures) on one instrument scale and as 3.5605 cm (five significant figures) on another. The latter reading is from an instrument with a finer scale (why?) and gives more information and reliability.

Zeros and the decimal point must be properly dealt with in determining the number of significant figures in a result. For example, how many significant figures does 0.0543 m have? What about 209.4 m and 2705.0 m? In such cases, the following rules are generally used to determine significance:

1. Zeros at the beginning of a number are not significant. They merely locate the decimal point. For example,

0.0543 m has three significant figures (5, 4, and 3).

2. Zeros within a number are significant. For example,

209.4 m has four significant figures (2, 0, 9, and 4).

3. Zeros at the end of a number after the decimal point are significant. For example,

2705.0 has five significant figures (2, 7, 0, 5, and 0).

Some confusion may arise with whole numbers that have one or more zeros at the end without a decimal point. Consider, for example, 300 kg, where the zeros (called trailing zeros) may or may not be significant. In such cases, it is not clear which zeros serve only to locate the decimal point and which are actually part of the measurement (and hence significant). That is, if the first zero from the left ($3\underline{0}0$ kg) is the estimated digit in the measurement, then only two digits are reliably known, and there are only two significant figures.

Similarly, if the last zero is the estimated digit ($30\underline{0}$ kg), then there are three significant figures. This ambiguity is be removed by using *scientific (powers of 10) notation*:

3.0×10^2 kg has two significant figures.

3.00×10^2 kg has three significant figures.

This procedure is also helpful in expressing the significant figures in large numbers. For example, suppose that the average distance from Earth to the Sun, 93,000,000 miles, is known to only four significant figures. This is easily expressed in powers of 10 notation: 9.300×10^7 mi.

D. Computations with Measured Values

Calculations are often performed with measured values, and error and uncertainty are "propagated" by the mathematical operations—for example, multiplication or division. That is, errors are carried through to the results by the mathematical operations.

The error can be better expressed by statistical methods; however, a widely used procedure for *estimating* the uncertainty of a mathematical result involves the use of significant figures.

The number of significant figures in a measured value gives an indication of the uncertainty or reliability of a measurement. Hence, you might expect that the result of a mathematical operation can be no more reliable than the quantity with the least reliability, or smallest number of significant figures, used in the calculation. That is, *reliability cannot be gained through a mathematical operation.*

It is important to report the results of mathematical operations with the proper number of significant figures. This is accomplished by using rules for (1) multiplication and division and (2) addition and subtraction. To obtain the proper number of significant figures, one rounds the results off. The general rules used for mathematical operations and rounding follow.

Significant Figures in Calculations

1. When multiplying and dividing quantities, leave as many significant figures in the answer as there are in the quantity with the least number of significant figures.

2. When adding or subtracting quantities, leave the same number of decimal places (rounded) in the answer as there are in the quantity with the least number of decimal places.

Rules for Rounding*

1. If the first digit to be dropped is less than 5, leave the preceding digit as is.

2. If the first digit to be dropped is 5 or greater, increase the preceding digit by one.

Notice that in this method, five digits (0, 1, 2, 3, and 4) are rounded down and five digits (5, 6, 7, 8, and 9) are rounded up.

What the rules for determining significant figures mean is that the result of a calculation can be no more accurate than the least accurate quantity used. That is, **you cannot gain accuracy in performing mathematical operations**.

These rules come into play frequently when doing mathematical operations with a hand calculator that may give a string of digits. ● Fig. 1.5 shows the result of the division of 374 by 29. The result must be rounded off to two significant figures—that is, to 13. (Why?)

*It should be noted that these rounding rules give an approximation of accuracy, as opposed to the results provided by more advanced statistical methods.

Figure 1.5 Insignificant figures. The calculator shows the result of the division operation 374/29. Because there are only two significant figures in the 29, a reported result should have no more than two significant figures, and the calculator display value should be rounded off to 13.

Example 1.3 Applying the rules.

Multiplication:

$$\underset{(2\ sf)}{2.5\text{ m}} \times \underset{(4\ sf)}{1.308\text{ m}} = \underset{(2\ sf)}{3.3\text{ m}^2}$$

Division:

$$\frac{\overset{(4\ sf)}{882.0\text{ s}}}{\underset{(3\ sf)}{0.245\text{ s}}} = 3600\text{ s} = 3.60 \times 10^3\text{ s}$$

(*represented to three significant figures; why*?)

Addition:

$$\begin{array}{r} 46.4 \\ 1.37 \\ 0.505 \\ \hline 48.275 \end{array} \longrightarrow 48.3$$

(*rounding off*)

(*46.4 has the least number of decimal places*)

Subtraction:

$$\begin{array}{r} 163 \\ -4.5 \\ \hline 158.5 \end{array} \rightarrow 159$$

(rounding off)

(*163 has the least number of decimal places, none*)

E. Expressing Experimental Error and Uncertainty

Percent Error

The object of some experiments is to determine the value of a well-known physical quantity—for example, the value of π.

The **accepted or "true" value** of such a quantity found in textbooks and physics handbooks is the most accurate value (usually rounded off to a certain number of significant figures) obtained through sophisticated experiments or mathematical methods.

The **absolute difference** between the experimental value E and the accepted value A, written $|E - A|$, is the *positive* difference in the values, for example, $|2 - 4| = |-2| = 2$ and $|4 - 2| = 2$. Simply subtract the smaller value from the larger, and take the result as positive. For a set of measurements, E is taken as the average value of the experimental measurements.

The **fractional error** is the ratio of the absolute difference and the accepted value:

$$\text{Fractional error} = \frac{\text{absolute difference}}{\text{accepted value}}$$

or

$$\boxed{\text{Fractional error} = \frac{|E - A|}{A}} \qquad \textbf{(1.1)}$$

The fractional error is commonly expressed as a percentage to give the **percent error** of an experimental value.*

$$\text{Percent error} = \frac{\text{absolute difference}}{\text{accepted value}} \times 100\%$$

or

$$\boxed{\text{Percent error} = \frac{|E - A|}{A} \times 100\%} \qquad \textbf{(1.2)}$$

Example 1.4 A cylindrical object is measured to have a diameter d of 5.25 cm and a circumference c of 16.38 cm. What are the experimental value of π and the percent error of the experimental value if the accepted value of π **to two decimal places** is 3.14?

Solution with $d = 5.25$ cm and $c = 16.38$ cm,

$$c = \pi d \quad \text{or} \quad \pi = \frac{c}{d} = \frac{16.38}{5.25} = 3.12$$

*It should be noted that percent error only gives a measure of experimental error or uncertainty when the accepted or standard value is highly accurate. If an accepted value itself has a large degree of uncertainty, then the percent error does not give a measure of experimental uncertainty.

Then $E = 3.12$ and $A = 3.14$, so

$$\text{Percent error} = \frac{|E - A|}{A} \times 100\%$$
$$= \frac{|3.12 - 3.14|}{3.14} \times 100\%$$
$$= \frac{0.02}{3.14} \times 100\% = 0.6\%$$

Note: To avoid rounding errors, the preferred order of operations is addition and subtraction before multiplication and division.*

If the uncertainty in experimentally measured values as expressed by the percent error is large, you should check for possible sources of error. If found, additional measurements should then be made to reduce the uncertainty. Your instructor may wish to set a maximum percent error for experimental results.

Percent Difference

It is sometimes instructive to compare the results of two measurements when there is no known or accepted value. The comparison is expressed as a **percent difference**, which is the ratio of the absolute difference between the experimental values E_2 and E_1 and the average or mean value of the two results, expressed as a percent.

$$\text{Percent difference} = \frac{\text{absolute difference}}{\text{average}} \times 100\%$$

or

$$\boxed{\text{Percent difference} = \frac{|E_2 - E_1|}{(E_2 + E_1)/2} \times 100\%} \qquad \textbf{(1.3)}$$

Dividing by the average or mean value of the experimental values is logical, because there is no way of deciding which of the two results is better.

Example 1.5 What is the percent difference between two measured values of 4.6 cm and 5.0 cm?

Solution With $E_1 = 4.6$ cm and $E_2 = 5.0$ cm,

$$\text{Percent difference} = \frac{|E_2 - E_1|}{(E_2 + E_1)/2} \times 100\%$$

$$\text{Percent difference} = \frac{|5.0 - 4.6|}{(5.0 + 4.6)/2} \times 100\%$$
$$= \frac{0.4}{4.8} \times 100\% = 8\%$$

As in the case of percent error, when the percent difference is large, it is advisable to check the experiment for errors and possibly make more measurements.

In many instances there will be more than two measurement values.

When there are three or more measurements, the percent difference is found by dividing the absolute value of the difference of the extreme values (that is, the values with greatest difference) by the average or mean value of all the measurements.

Average (Mean) Value

Most experimental measurements are repeated several times, and it is very unlikely that identical results will be obtained for all trials. For a set of measurements with predominantly random errors (that is, the measurements are all equally trustworthy or probable), it can be shown mathematically that the true value is most probably given by the average or mean value.

The **average** or **mean value** $\bar{x}$ of a set of N measurements is

$$\boxed{\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i} \qquad \textbf{(1.4)}$$

where the summation sign Σ is a shorthand notation indicating the sum of N measurements from x_1 to x_N. ($\bar{x}$ is commonly referred to simply as the *mean.*)

Example 1.6 What is the average or mean value of the set of numbers 5.42, 6.18, 5.70, 6.01, and 6.32?

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$
$$= \frac{5.42 + 6.18 + 5.70 + 6.01 + 6.32}{5}$$
$$= 5.93$$

There are other, more advanced methods to express the dispersion or precision of sets of measurements. Two of these are given in the appendices. Appendix C: "Absolute Deviation from the Mean and Mean Absolute Deviation," and Appendix D: "Standard Deviation and Method of Least Squares."

*Although percent error is generally defined using the absolute difference $|E - A|$, some instructors prefer to use $(E - A)$, which results in positive (+) or negative (−) percent errors, for example, −0.6% in Example 1.4. In the case of a series of measurements and computed percent errors, this gives an indication of systematic error.

F. Graphical Representation of Data

It is often convenient to represent experimental data in graphical form, not only for reporting but also to obtain information.

Graphing Procedures

Quantities are commonly plotted using rectangular Cartesian axes (X and Y). The horizontal axis (X) is called the *abscissa,* and the vertical axis (Y), the *ordinate*. The location of a point on the graph is defined by its coordinates x and y, written (x, y), referenced to the origin O, the intersection of the X and Y axes.

When plotting data, choose axis scales that are easy to plot and read. The graph in ● Fig. 1.6A shows an example of scales that are too small. This "bunches up" the data, making the graph too small, and the major horizontal scale values make it difficult to read intermediate values. Also, the dots or data points should not be connected. Choose scales so that most of the graph paper is used. The graph in ● Fig. 1.6B shows data plotted with more appropriate scales.*

Also note in Fig. 1.6A that scale units on the axes are not given. For example, you don't know whether the units of displacement are feet, meters, kilometers, or whatever. *Scale units should always be included,* as in Fig. 1.6B. It is also acceptable, and saves time, to use standard unit abbreviations, such as N for newton and m for meter. This will be done on subsequent graphs.

With the data points plotted, draw a smooth line described by the data points. *Smooth* means that the line does not have to pass exactly through each point but connects the general areas of significance of the data points (*not* connecting the data points as in Fig. 1.6A). The graph

*As a general rule, it is convenient to choose the unit of the first major scale division to the right or above the origin or zero point as 1, 2, or 5 (or multiples or submultiples thereof, for example, 10 or 0.1) so that the minor (intermediate) scale divisions can be easily interpolated and read.

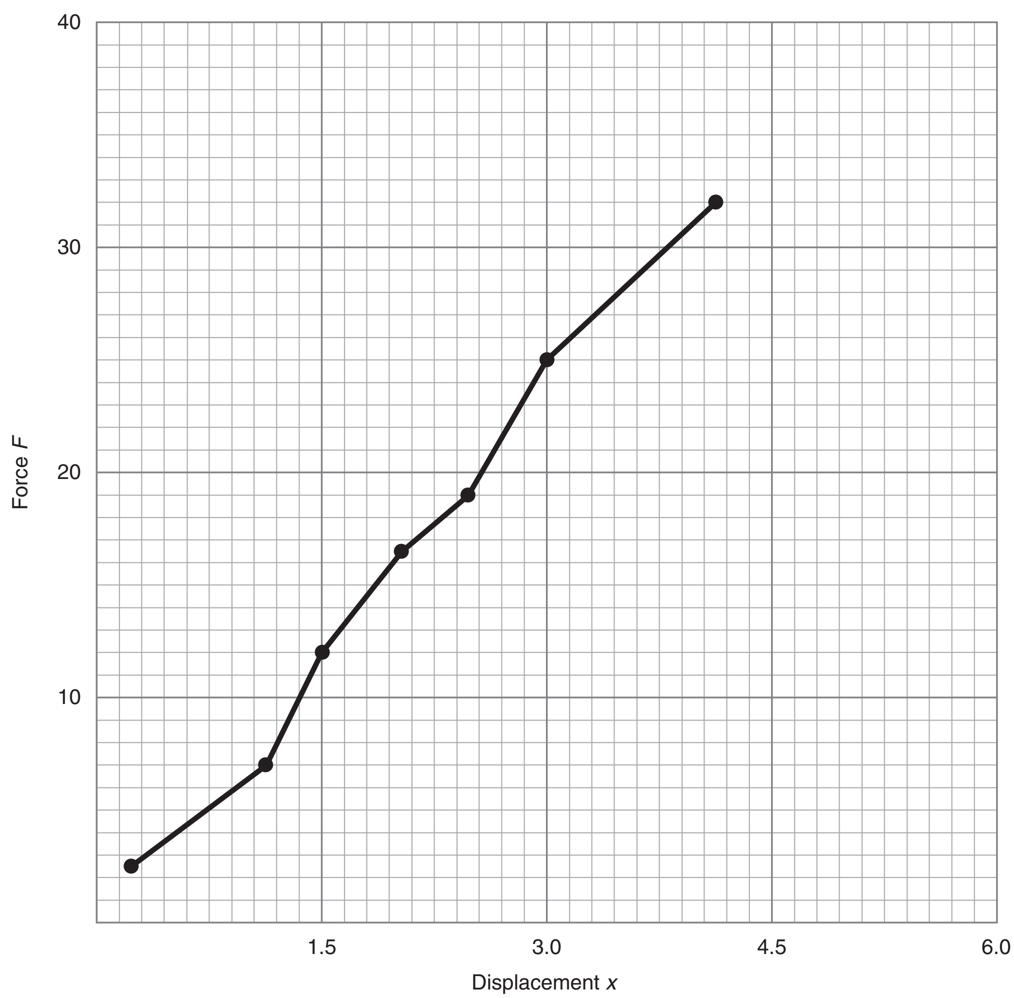


Figure 1.6A Poor graphing. An example of an improperly labeled and plotted graph. See text for description.

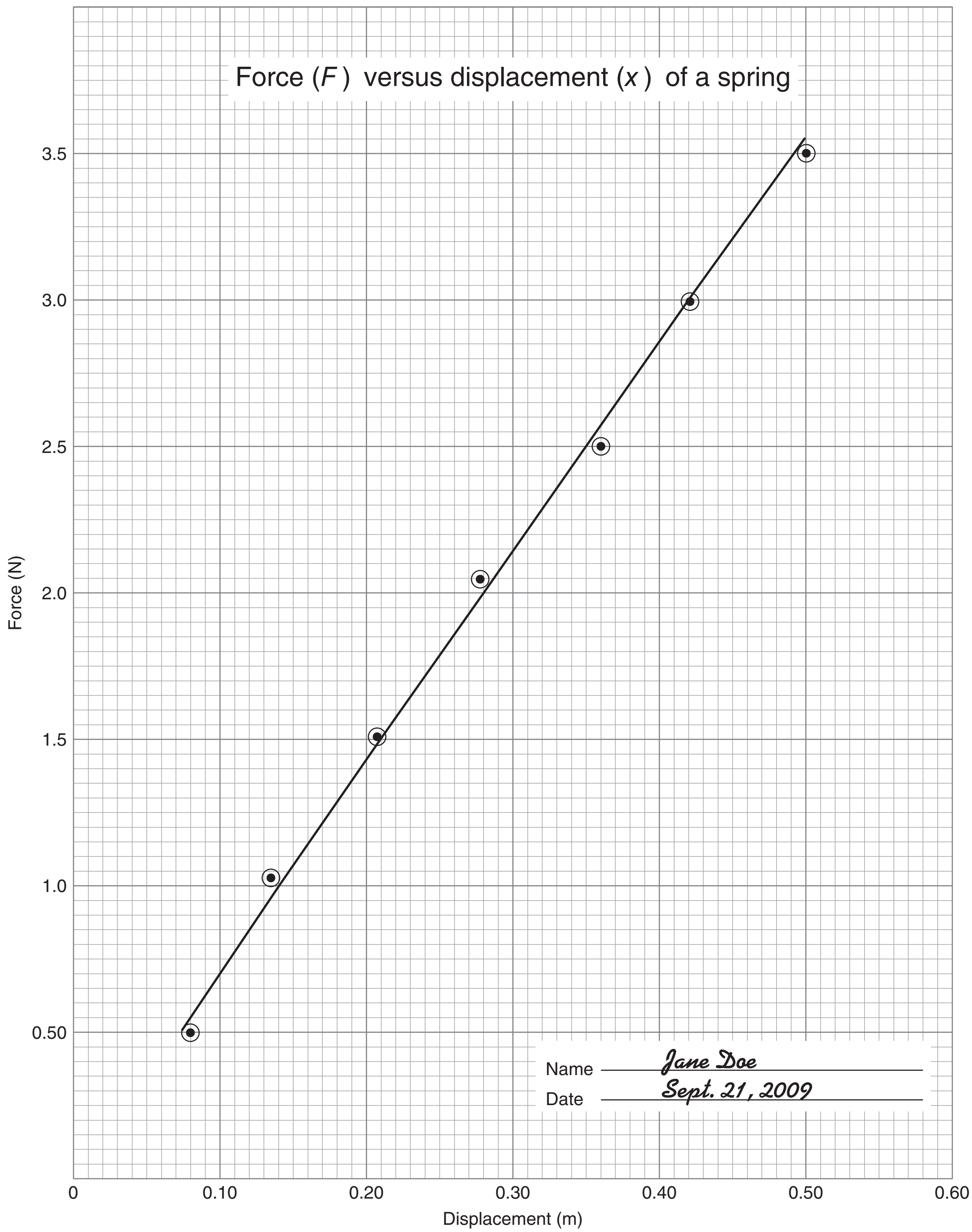


Figure 1.6B Proper graphing. An example of a properly labeled and plotted graph. See text for description.

TABLE 1.1 Data for Figure 1.7

Mass (kg)	Period (s)	±	$\bar{d}$
0.025	1.9	±	0.40
0.050	2.7	±	0.30
0.10	3.8	±	0.25
0.15	4.6	±	0.28
0.20	5.4	±	0.18
0.25	6.0	±	0.15

in Fig. 1.6B with an approximately equal number of points on each side of the line gives a "line of best fit."†

In cases where several determinations of each experimental quantity are made, the average value is plotted and the mean deviation or the standard deviation may be plotted as *error bars*. For example, the data for the period of a mass oscillating on a spring given in Table 1.1 are plotted in ● Fig. 1.7, period (T) versus mass (m). (The $\bar{d}$ is the mean deviation, shown here for an illustration of error bars. See Appendix C.)* A smooth line is drawn so as to pass within the error bars. (Your instructor may want to explain the use of a French curve at this point.)

Graphs should have the following elements (see Fig. 1.7):

1. Each axis labeled with the quantity plotted.
2. The units of the quantities plotted.
3. The title of the graph on the graph paper (commonly listed as the y-coordinate versus the x-coordinate).
4. Your name and the date.

STRAIGHT-LINE GRAPHS

Two quantities (x and y) are often linearly related; that is, there is an algebraic relationship of the form $y = mx + b$, where m and b are constants. When the values of such quantities are plotted, the graph is a straight line, as shown in ● Fig. 1.8.

The m in the algebraic relationship is called the **slope** of the line and is equal to the ratio of the intervals $\Delta y/\Delta x$. Any set of intervals may be used to determine the slope of a straight-line graph; for example, in Fig. 1.8,

$$m = \frac{\Delta y_1}{\Delta x_1} = \frac{15\text{ cm}}{2.0\text{ s}} = 7.5\text{ cm/s}$$

$$m = \frac{\Delta y_2}{\Delta x_2} = \frac{45\text{ cm}}{6.0\text{ s}} = 7.5\text{ cm/s}$$

†The straight line of "best fit" for a set of data points on a graph can be determined by a statistical procedure called *linear regression*, using what is known as the *method of least squares*. This method determines the best-fitting straight line by means of differential calculus, which is beyond the scope of this manual. The resulting equations are given in Appendix D, along with the procedure for determining the slope and intercept of a best-fitting straight line.

*The mean deviation and standard deviation are discussed in Appendix C and D, respectively. They give an indication of the dispersion of a set of measured values. These methods are optional at your instructor's discretion.

Points should be chosen relatively far apart on the line. For best results, points corresponding to data points should not be chosen, even if they appear to lie on the line.

The b in the algebraic relationship is called the **y-intercept** and is equal to the value of the y-coordinate where the graph line intercepts the Y-axis. In Fig. 1.8, b =3 cm. Notice from the relationship that $y = mx + b$, so that when $x = 0$, then $y = b$. If the intercept is at the origin (0, 0), then $b = 0$.

The equation of the line in the graph in Fig. 1.8 is $d = 7.5t + 3$. The general equation for uniform motion has the form $d = vt + d_o$. Hence, the initial displacement $d_o = 3$ cm and the speed $v = 7.5$ cm/s.

Some forms of nonlinear functions that are common in physics can be represented as straight lines on a Cartesian graph. This is done by plotting nonlinear values. For example, if

$$y = ax^2 + b$$

is plotted on a regular y-versus-x graph, a parabola would be obtained. But if $x^2 = x'$ were used, the equation becomes

$$y = ax' = b$$

which has the form of a straight line.

This means plotting y versus x', would give a straight line. Since $x' = x^2$, the squared values of x must be plotted. That is, square all the values of x in the data table, and plot these numbers with the corresponding y values.

Other functions can be "straightened out" by this procedure, including an exponential function:

$$y = Ae^{ax}$$

In this case, taking the natural logarithm of both sides:

$$\ln y = \ln A + \ln e^{ax}$$

or

$$\ln y = ax + \ln A$$

(where $\ln e^x = x$)
Plotting the values of the natural (base e) logarithm versus x gives a straight line with slope a and an intercept $\ln A$.

Similarly, for

$$y = ax^n$$

using the common (base 10) logarithm,

$$\log y = \log a + \log x^n$$

and

$$\log y = n \log x + \log a$$

(where $\log x^n = n \log x$).

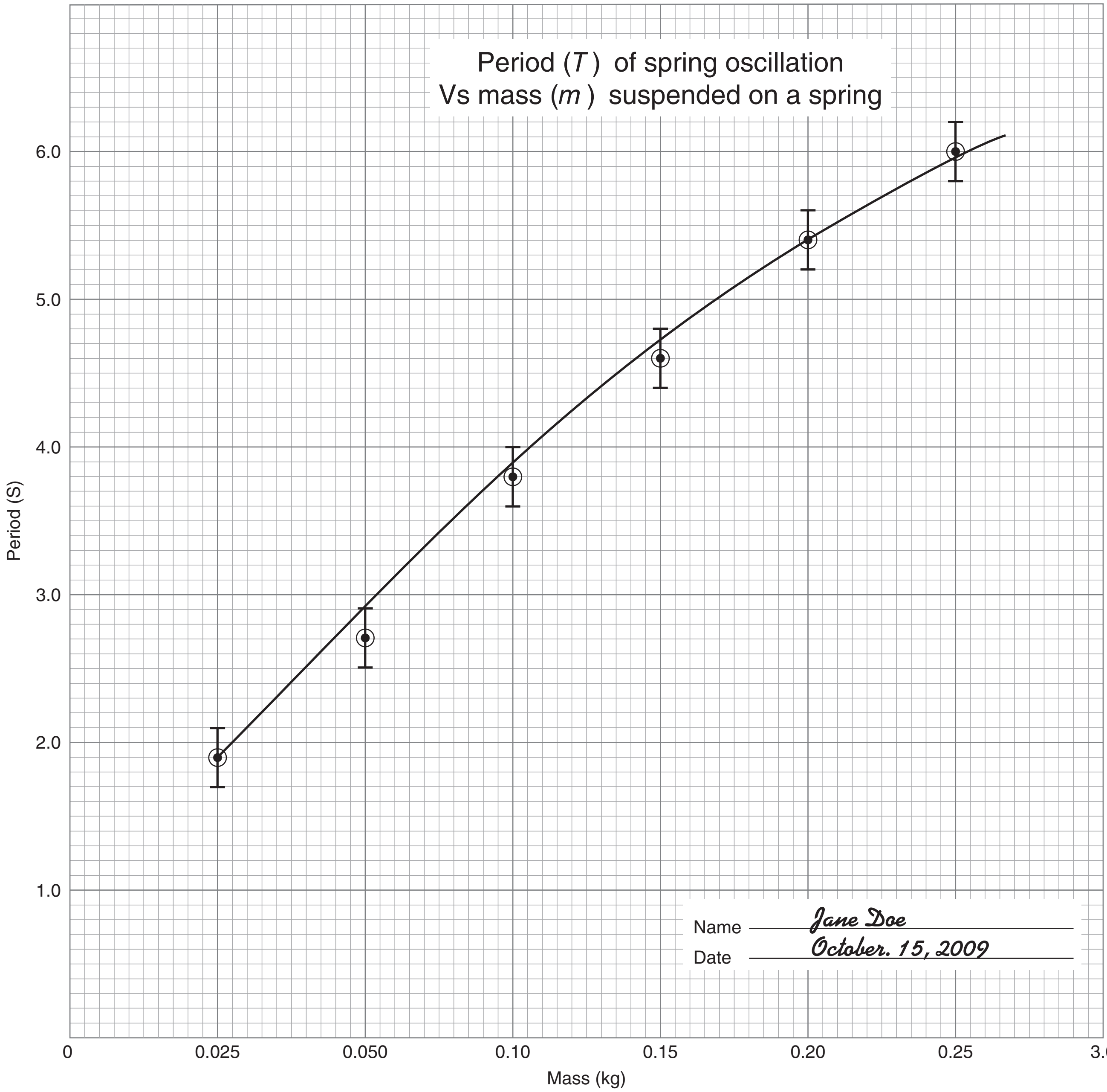


Figure 1.7 Error bars. An example of graphically presented data with error bars. An error bar indicates the precision of a measurement. In this case, the error bars represent mean deviations.

Plotting the values of log y versus log x gives a straight line with slope n and intercept log a. (See Appendix E.)

EXPERIMENTAL PROCEDURE

Complete the exercises in the Laboratory Report, showing calculations and attaching graphs as required. (*Note:* In this experiment and throughout, attach an additional sheet for calculations if necessary.)

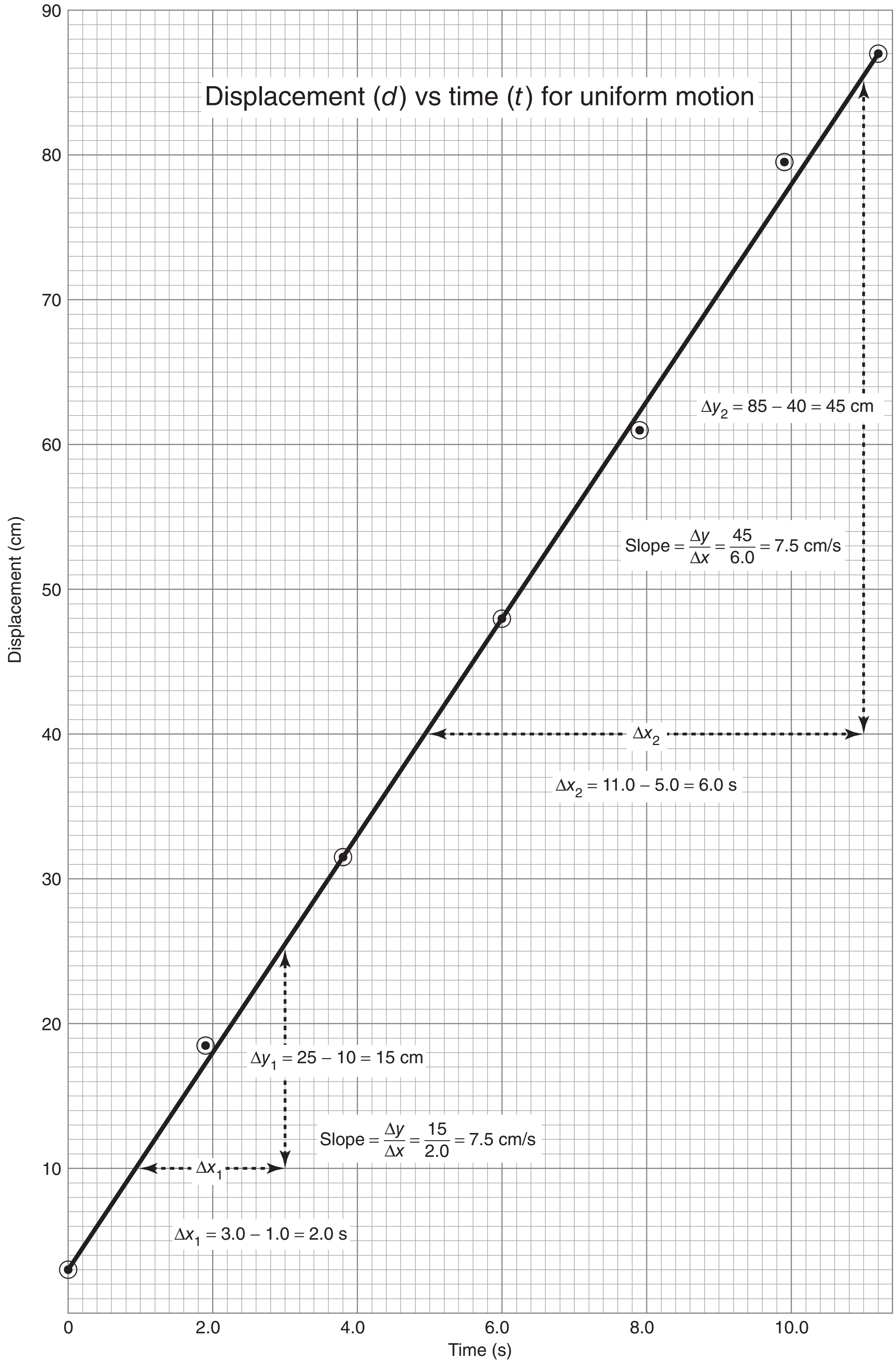


Figure 1.8 Straight-line slope. Examples of intervals for determining the slope of a straight line. The slope is the ratio of $\Delta y/\Delta x$ (or $\Delta d/\Delta t$). Any set of intervals may be used, but the endpoints of an interval should be relatively far apart, as for $\Delta y_2/\Delta x_2$.

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Name ______________________ Section ______________ Date ______________

Lab Partner(s) ____

E X P E R I M E N T 1

Experimental Uncertainty (Error) and Data Analysis

TI *Laboratory Report*

1. Least Counts
   (a) Given meter-length sticks calibrated in meters, decimeters, centimeters, and millimeters, respectively. Use the sticks to measure the length of the object provided and record with the appropriate number of significant figures in Data Table 1.

DATA TABLE 1

Purpose: To express least counts and measurements.

Object Length			
m	dm	cm	mm

Actual length ______________
(Provided by instructor after measurements)
Comments on the measurements in terms of least counts:

   (b) Find the percent errors for the four measurements in Data Table 1.

DATA TABLE 2

Purpose: To express the percent errors.

	Object Length
Least Count	
% Error	

Comments on the percent error results:

2. Significant Figures
   (a) Express the numbers listed in Data Table 3 to three significant figures, writing the numbers in the first column in normal notation and the numbers in the second column in powers of 10 (scientific) notation.

(continued)

DATA TABLE 3

Purpose: To practice expressing significant figures.

0.524	________	5280	________
15.08	________	0.060	________
1444	________	82.453	________
0.0254	________	0.00010	________
83,909	________	2,700,000,000	________

(b) A rectangular block of wood is measured to have the dimensions 11.2 cm × 3.4 cm × 4.10 cm. Compute the volume of the block, showing explicitly (by underlining) how doubtful figures are carried through the calculation, and report the final answer with the correct number of significant figures.

Calculations
(show work)

Computed volume
(in powers of 10 notation) ________________
(units)

(c) In an experiment to determine the value of π, a cylinder is measured to have an average value of 4.25 cm for its diameter and an average value of 13.39 cm for its circumference. What is the experimental value of π to the correct number of significant figures?

Calculations
(show work)

Experimental value of π ________________
(units)

Name ______________________ Section ______________ Date ______________

Lab Partner(s) ______________________

3. Expressing Experimental Error
   (a) If the accepted value of π is 3.1416, what are the fractional error and the percent error of the experimental value found in 2(c)?

Calculations
(show work)

Fractional error ______________

Percent error ______________

   (b) In an experiment to measure the acceleration g due to gravity, two values, 9.96 m/s^2 and 9.72 m/s^2, are determined. Find (1) the percent difference of the measurements, (2) the percent error of each measurement, and (3) the percent error of their mean. (Accepted value: $g = 9.80$ m/s^2.)

Calculations
(show work)

Percent difference ______________

Percent error of E_1 ______________

Percent error of E_2 ______________

Percent error of mean ______________

(continued)

(c) Data Table 4 shows data taken in a free-fall experiment. Measurements were made of the distance of fall (y) at each of four precisely measured times. Complete the table. Use only the proper number of significant figures in your table entries, even if you carry extra digits during your intermediate calculations.

DATA TABLE 4

Purpose: To practice analyzing data.

Time t	Distance (m)						(Optional)	t^2
(s)	y_1	y_2	y_3	y_4	y_5	$\bar{y}$	$\bar{d}$	( )
0	0	0	0	0	0			
0.50	1.0	1.4	1.1	1.4	1.5			
0.75	2.6	3.2	2.8	2.5	3.1			
1.00	4.8	4.4	5.1	4.7	4.8			
1.25	8.2	7.9	7.5	8.1	7.4			

(d) Plot a graph of $\bar{y}$ versus t (optional: with $2\bar{d}$ error bars) for the free-fall data in part (c). Remember that $t = 0$ is a known point.

(e) The equation of motion for an object in free fall starting from rest is $y = \frac{1}{2}gt^2$, where g is the acceleration due to gravity. This is the equation of a parabola, which has the general form $y = ax^2$.

Convert the curve into a straight line by plotting $\bar{y}$ versus t^2. That is, plot the square of the time on the abscissa. Determine the slope of the line and compute the experimental value of g from the slope value.

Calculations
(show work)

Experimental value of g from graph ______________
(units)

(f) Compute the percent error of the experimental value of g determined from the graph in part (e). (Accepted value: $g = 9.8\ \text{m/s}^2$.)

Calculations
(show work)

Percent error ______________

(g) The relationship of the applied force F and the displacement x of a spring has the general form $F = kx$, where the constant k is called the *spring constant* and is a measure of the "stiffness" of the spring. Notice that this equation has the form of a straight line. Find the value of the spring constant k of the spring used in determining the experimental data plotted in the Fig. 1.6B graph. (*Note:* Because $k = F/x$, the units of k in the graph are N/m.)

Calculations
(show work)

Value of spring constant of
spring in Fig. 1.6B graph ______________
(units)

(h) The general relationship of the period of oscillation T of a mass m suspended on a spring is $T = 2\pi\sqrt{m/k}$, where k is the spring constant. Replot the data in Fig. 1.7 so as to obtain a straight-line graph, and determine the value of the spring constant used in the experiment. [*Hint:* Square both sides of the equation, and plot in a manner similar to that used in part (e).] Show the final form of the equation and calculations.

Calculations
(show work)

Value of spring constant of
spring in Fig. 1.7 ______________
(units)

(i) The data in sections (g) and (h) above were for the same spring. Compute the percent difference for the values of the spring constants obtained in each section.

(continued)

TI QUESTIONS

1. Read the measurements on the rulers in ● Fig. 1.9, and comment on the results.

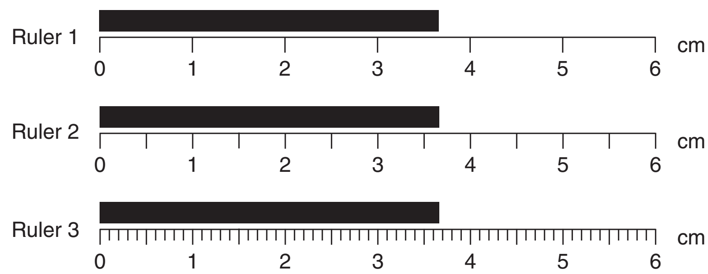

Figure 1.9

2. Were the measurements of the block in part (b) of Procedure 2 all done with the same instrument? Explain.

3. Referring to the dart analogy in Fig. 1.3, draw a dart grouping that would represent poor precision but good accuracy with an average value.

4. Do percent error and percent difference give indications of accuracy or precision? Discuss each.

5. Suppose you were the first to measure the value of some physical constant experimentally. How would you provide an estimate of the experimental uncertainty?